43.48.

$$\iint\limits_S z\cdot dS = \iint\limits_\Omega v\cdot\sqrt{E\cdot G-F^2}\cdot du\cdot dv =$$

$x = u\cdot\cos v$
$y = u\cdot\sin v$
$z = v$
$0 < u < a$
$0 < v < 2\pi$

$$E = \left(\frac{\partial x}{\partial u}\right)^2+\left(\frac{\partial y}{\partial u}\right)^2+\left(\frac{\partial z}{\partial u}\right)^2 =$$

$$= \left(\frac{\partial(u\cdot\cos v)}{\partial u}\right)^2+\left(\frac{\partial(u\cdot\sin v)}{\partial u}\right)^2+\left(\frac{\partial v}{\partial u}\right)^2 =$$

$$= (\cos v\cdot 1)^2+(\sin v\cdot 1)^2+0^2 = \cos^2 v+\sin^2 v+0 = 1$$

$$G = \left(\frac{\partial x}{\partial v}\right)^2+\left(\frac{\partial y}{\partial v}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2 = \left(\frac{\partial(u\cdot\cos v)}{\partial v}\right)^2+\left(\frac{\partial(u\cdot\sin v)}{\partial v}\right)^2+\left(\frac{\partial v}{\partial v}\right)^2 =$$

$$= (u\cdot(-\sin v))^2+(u\cdot\cos v)^2+1^2 = u^2\cdot(\sin^2 v+\cos^2 v)+1 = u^2+1$$

$$F = \frac{\partial x}{\partial u}\cdot\frac{\partial x}{\partial v}+\frac{\partial y}{\partial u}\cdot\frac{\partial y}{\partial v}+\frac{\partial z}{\partial u}\cdot\frac{\partial z}{\partial v} =$$

$$= \frac{\partial(u\cdot\cos v)}{\partial u}\cdot\frac{\partial(u\cdot\cos v)}{\partial v}+\frac{\partial(u\cdot\sin v)}{\partial u}\cdot\frac{\partial(u\cdot\sin v)}{\partial v}+\frac{\partial v}{\partial u}\cdot\frac{\partial v}{\partial v} =$$

$$= \cos v\cdot 1\cdot u\cdot(-\sin v)+\sin v\cdot 1\cdot u\cdot\cos v+0\cdot 1 =$$

$$= -u\cdot\sin v\cdot\cos v+u\cdot\sin v\cdot\cos v+0 = 0+0 = 0$$

$$= \iint\limits_\Omega v\cdot\sqrt{1\cdot(u^2+1)-0^2}\cdot du\cdot dv = \iint\limits_\Omega\sqrt{u^2+1}\cdot v\cdot du\cdot dv =$$

$$= \int_0^{2\pi} v\cdot dv\cdot\int_0^a\sqrt{u^2+1}\cdot du = \left.\frac{v^2}{2}\right|_0^{2\pi}\cdot\left[\frac{u}{2}\cdot\sqrt{u^2+1}+\frac{1}{2}\cdot\ln\left(u+\sqrt{u^2+1}\right)\right]_0^a =$$

$$= \left[\frac{(2\pi)^2}{2}-\frac{0^2}{2}\right]\cdot\left[\left(\frac{a}{2}\cdot\sqrt{a^2+1}+\frac{1}{2}\cdot\ln\left(a+\sqrt{a^2+1}\right)\right)-\left(\frac{0}{2}\cdot\sqrt{0^2+1}+\frac{1}{2}\cdot\ln\left(0+\sqrt{0^2+1}\right)\right)\right] =$$

$$= \left[\frac{4\pi^2}{2}-0\right]\cdot\left[\frac{a}{2}\cdot\sqrt{a^2+1}+\frac{1}{2}\cdot\ln\left(a+\sqrt{a^2+1}\right)-\left(0+\frac{1}{2}\cdot\ln 1\right)\right] =$$

$$= 2\pi^2\cdot\frac{1}{2}\cdot\left[a\cdot\sqrt{a^2+1}+\ln\left(a+\sqrt{a^2+1}\right)-\frac{1}{2}\cdot 0\right] =$$

$$= \pi^2\cdot\left(a\cdot\sqrt{a^2+1}+\ln\left(a+\sqrt{a^2+1}\right)\right).$$

43.49.

$$\iint\limits_S z^2\cdot dS = \iint\limits_\Omega r^2\cdot\cos^2\alpha\cdot\sqrt{E\cdot G-F^2}\cdot dr\cdot d\varphi =$$

$x = r\cdot\cos\varphi\cdot\sin\alpha$
$y = r\cdot\sin\varphi\cdot\sin\alpha$
$z = r\cdot\cos\alpha$
$0\le r\le a$
$0\le\varphi\le 2\pi$
$\alpha = \text{const}\ \left(0<\alpha<\frac{\pi}{2}\right)$

$$E = \left(\frac{\partial x}{\partial r}\right)^2+\left(\frac{\partial y}{\partial r}\right)^2+\left(\frac{\partial z}{\partial r}\right)^2 =$$

$$=\left(\frac{\partial(r\cdot\cos\varphi\cdot\sin\alpha)}{\partial r}\right)^2+\left(\frac{\partial(r\cdot\sin\varphi\cdot\sin\alpha)}{\partial r}\right)^2+\left(\frac{\partial(r\cdot\cos\alpha)}{\partial r}\right)^2=$$

$$=(\cos\varphi\cdot\sin\alpha)^2+(\sin\varphi\cdot\sin\alpha)^2+(\cos\alpha)^2=(\cos^2\varphi+\sin^2\varphi)\cdot\sin^2\alpha+\cos^2\alpha=1$$

$$G=\left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2+\left(\frac{\partial z}{\partial\varphi}\right)^2=\left(\frac{\partial(r\cdot\cos\varphi\cdot\sin\alpha)}{\partial\varphi}\right)^2+\left(\frac{\partial(r\cdot\sin\varphi\cdot\sin\alpha)}{\partial\varphi}\right)^2+\left(\frac{\partial(r\cdot\cos\alpha)}{\partial\varphi}\right)^2=$$

$$=\left(r\cdot\sin\alpha\cdot(-\sin\varphi)\right)^2+(r\cdot\sin\alpha\cdot\cos\varphi)^2+0^2=r^2\cdot\sin^2\alpha\cdot(\sin^2\varphi+\cos^2\varphi)=r^2\cdot\sin^2\alpha$$

$$F=\frac{\partial x}{\partial r}\cdot\frac{\partial x}{\partial\varphi}+\frac{\partial y}{\partial r}\cdot\frac{\partial y}{\partial\varphi}+\frac{\partial z}{\partial r}\cdot\frac{\partial z}{\partial\varphi}=$$

$$=\frac{\partial(r\cdot\cos\varphi\cdot\sin\alpha)}{\partial r}\cdot\frac{\partial(r\cdot\cos\varphi\cdot\sin\alpha)}{\partial\varphi}+\frac{\partial(r\cdot\sin\varphi\cdot\sin\alpha)}{\partial r}\cdot\frac{\partial(r\cdot\sin\varphi\cdot\sin\alpha)}{\partial\varphi}+$$

$$+\frac{\partial(r\cdot\cos\alpha)}{\partial r}\cdot\frac{\partial(r\cdot\cos\alpha)}{\partial\varphi}=$$

$$=\cos\varphi\cdot\sin\alpha\cdot r\cdot(-\sin\varphi)\cdot\sin\alpha+\sin\varphi\cdot\sin\alpha\cdot r\cdot\cos\varphi\cdot\sin\alpha+\cos\alpha\cdot 0=$$

$$=-r\cdot\cos\varphi\cdot\sin\varphi\cdot\sin^2\alpha+r\cdot\sin\varphi\cdot\cos\varphi\cdot\sin^2\alpha+0=0+0=0$$

$$=\iint\limits_\Omega r^2\cdot\cos^2\alpha\cdot\sqrt{1\cdot r^2\cdot\sin^2\alpha-0^2}\cdot dr\cdot d\varphi=\iint\limits_\Omega r^2\cdot\cos^2\alpha\cdot\sqrt{r^2\sin^2\alpha}\cdot dr\cdot d\varphi=$$

$$=\iint\limits_\Omega r^2\cdot\cos^2\alpha\cdot r\cdot\sin\alpha\cdot dr\cdot d\varphi=\iint\limits_\Omega r^3\cdot\sin\alpha\cdot\cos^2\alpha\cdot dr\cdot d\varphi=$$

$$=\sin\alpha\cdot\cos^2\alpha\cdot\iint\limits_\Omega r^3\cdot dr\cdot d\varphi=\sin\alpha\cdot\cos^2\varphi\cdot\int_0^{2\pi}d\varphi\cdot\int_0^a r^3\cdot dr=$$

$$=\sin\alpha\cdot\cos^2\alpha\cdot\varphi\Big|_0^{2\pi}\cdot\frac{r^4}{4}\Big|_0^{\alpha}=\sin\alpha\cdot\cos^2\alpha\cdot(2\pi-0)\cdot\left(\frac{a^4}{4}-\frac{0^4}{4}\right)=$$

$$=\sin\alpha\cdot\cos^2\alpha\cdot 2\pi\cdot\frac{a^4}{4}=\frac{\pi\cdot a^4}{2}\cdot\sin\alpha\cdot\cos^2\alpha.\qquad\left(0<\alpha<\frac{\pi}{2}\right)$$