$N^{\underline{o}}$ 1Б

$$V(r) = \frac{e^2}{r} + \frac{a}{r^2}$$

$$-\frac{\hbar^2}{2\mu}\Delta\psi + V(r)\psi(r) = E\psi$$

$$\left\{-\frac{\hbar^2}{2\mu}\Delta + \frac{e^2}{r} + \frac{a}{r^2}\right\}\psi(r) = E\psi(r)$$

$$\Delta = \Delta_r + \frac{1}{r^2}\Delta_{\theta,\varphi} \qquad [H, L^2] = 0$$

$$L^2 = -\hbar^2\Delta_{\theta,\varphi}$$

$$\psi = R(r)\,Y_{lm}(\theta,\varphi)$$

$$-\frac{\hbar^2}{2\mu}\left\{Y_{lm}\Delta_r R + \frac{R}{r^2}\Delta_{\theta,\varphi}Y_{lm}\right\} + \left(-\frac{e^2}{r} + \frac{a}{r^2}\right)R = E\,RY(\theta,\varphi)$$

$$l = \overline{0, n-1} \qquad m = -l, l \;, \quad E_n = -\frac{R}{n^2}$$

$$\sum_{l=0}\sum_{m=-1} 1 = \sum_{l}(2l+1) = \frac{1+2n-1}{2}n = n^2 \qquad R = \frac{\mu e^4}{2\hbar^2}$$

$$-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - l(l+1)\right]R + \left(\frac{-e^2}{r} + \frac{a}{r^2} - E\right)R = 0$$

$$-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{l(l+1) + \frac{2\mu}{\hbar}a}{r^2}R\right] = ER$$

$$-\frac{\hbar^2}{2\mu}\Delta_r R + \left(\frac{\hbar^2}{2\mu}l(l+1)\frac{1}{r^2} - \frac{e^2}{r} - E\right)R = 0$$

$$-l(l+1) + \frac{2\mu}{\hbar^2}a = S(S+1)$$

$$S = \frac{-1 + (2l+1)\sqrt{1 + 8\pi C/\hbar^2(2l+1)^2}}{2}, \quad n = S + l + 1$$

$$E_n = \frac{R}{(l+S+1)^2} = \frac{\mu e^4}{2\hbar^2 n^2}$$

Розрахувати для основного стану атома водню середні значення $\langle r \rangle$ та $\langle r^2 \rangle$ №17

$$\psi = c e^{-r/a} \qquad a = \frac{\hbar^2}{zme^2} -$$

a - борівський радіус

ψ_{nem} $n=1,\ l=m=0$

$$\psi_{100} = c e^{-r/a}$$

$$\int_0^\infty \psi_{100}^* \psi_{100}\, dr = 1$$

$$4\pi \int_0^\infty |c|^2 r^2 e^{-\frac{2r}{a}} dr = 4\pi c^2 \int_0^\infty r^2 e^{-\frac{2r}{a}} =$$

$$= \left| \frac{2r}{a} = r' \Rightarrow \frac{ar'}{2} = r \right| \; dr = \frac{a}{2} dr' = 4\pi c^2 \int_0^\infty \frac{a^2}{4} r'^2 e^{-r'} \frac{a}{2} dr' =$$

$$= 4\pi c^2 \int_0^\infty \frac{a^3}{8} r'^2 e^{-r'} dr' = \pi a^3 c^2 = 1 \Rightarrow$$

$$\Rightarrow c = \frac{1}{\sqrt{\pi a^3}}$$

$$\langle r \rangle = \int \psi^* \psi\, r\, dr = \int |\psi|^2 r\, dr =$$

$$= \frac{1}{\pi a^3} \int_0^\infty e^{-2r/a} r^3 dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi =$$

$$= \frac{4\pi}{\pi a^3} \int_0^\infty r^3 e^{-\frac{2r}{a}} dr = \frac{4}{a^3} \int_0^\infty e^{-\frac{2r}{a}} r^3 dr =$$

$$= \begin{bmatrix} 2\frac{r}{a} = z \\ dr = \frac{a}{2} dz \end{bmatrix} = \frac{4}{a^3} \cdot \frac{a^3}{8} \cdot \frac{a}{2} \int_0^\infty e^{-z} z^3 dz =$$

$r^3 \Rightarrow \frac{a^3}{8}$

$$= \frac{a}{4} \int_0^\infty e^{-z} z^3 dz = \left[\int_0^\infty z^n e^{-z} dz = \cancel{\Gamma(n) = (n-1)!}\ n! \right] =$$

$$= \frac{a}{4} \cdot 3! = \frac{3}{2} a$$

$$\langle r^2 \rangle = \int_0^\infty |\psi|^2 r^2 dr = \frac{4\pi}{\pi a^3} \int_0^\infty r^4 e^{-\frac{2r}{a}} dr =$$

$$= \begin{bmatrix} \frac{ar'}{2} = r \\ dr = \frac{a}{2} dr' \end{bmatrix} = \frac{4}{\cancel{a^3}} \cdot \frac{a^4}{16} \cdot \frac{a}{2} \int_0^\infty r^4 e^{-r'} dr' =$$

$$= \frac{a^2}{8} 4! = \underline{3a^2}$$

№18

$$-\frac{\hbar^2}{2M}\frac{d^2\psi}{dx^2}+V(x)\psi=E\psi$$

I Припускаємо що рівні енергії вирод.

$$-\frac{\hbar^2}{2M}\psi_1''+V(x)\psi_1(x)=E_1\psi_1(x)$$

$$-\frac{\hbar^2}{2M}\psi_2''+V(x)\psi_2(x)=E_2\psi_2(x)$$

$$\psi_2\psi_1''-\psi_2''\psi_1=W(\psi_1,\psi_2)=const$$

при $\psi\to\infty$ $const=0$, отже при всіх інших $W=0\Rightarrow\psi_1,\psi_2\to$ ЛЗ

спектр не вироджений

Припущення не вірне

2) $V(x)\to 0$ $x\to\pm\infty$

$$-\frac{\hbar^2}{2M}\frac{d^2\psi}{dx^2}=E\psi(x)$$

$$\frac{d^2\psi_\infty}{dx^2}=-\frac{2M}{\hbar^2}E\psi(x)$$

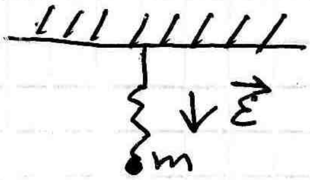
$$\psi_\infty''=-k^2\psi_\infty(x)$$

$$\psi_\infty(x)=Ae^{ikx}+\tilde{A}e^{-ikx}$$

$$E_k=\frac{\hbar^2k^2}{2m}$$ (2 кратне виродження)

$$W=\begin{vmatrix} e^{ikx} & e^{-ikx} \\ i\hbar e^{ikx} & i\hbar e^{-ikx}\end{vmatrix}=-2ik\neq 0$$ – ЛНЗ

№19



$$\vec F = e\vec{\mathcal{E}}$$

$$F_x = e\mathcal{E}$$

$$F = -\nabla U$$

$$F_x = -\frac{dU}{dx} \qquad e\mathcal{E} = -\frac{dU}{dx} \Rightarrow$$

$$\Rightarrow dU = e\mathcal{E}\,dx$$

$$-\frac{\hbar^2}{2M}\frac{d^2\psi}{dx^2} + \left(\frac{m\omega^2 x^2}{2} - e\mathcal{E}x\right)\psi = E\psi$$

$$\frac{m\omega^2 x^2}{2} - e\mathcal{E}x = \frac{m\omega^2}{2}\left(x^2 - \frac{2e\mathcal{E}x}{m\omega^2}\right) =$$

$$= \frac{m\omega^2}{2}\Big(\underbrace{x - \frac{\mathcal{E}e}{m\omega^2}}_{x'}\Big)^2 - \frac{e^2\mathcal{E}^2}{m^2\omega^4}$$

$$-\frac{\hbar^2}{2M}\frac{d^2\psi}{dx^2} + \frac{m\omega^2 x'^2}{2} = \underbrace{\left(E + \frac{e^2\mathcal{E}^2}{m\omega^2}\right)}_{E'}\psi$$

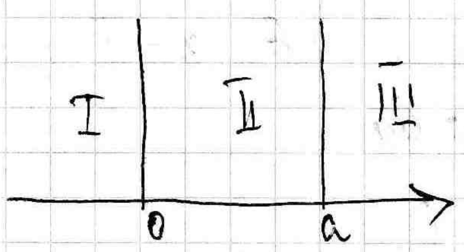
$$\psi_n(\xi) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}\,\alpha}}\, e^{-\frac{\xi^2}{2}} H_n(\xi), \quad \alpha = \sqrt{\frac{\hbar}{m\omega}}$$

$$\xi = \frac{x'}{\alpha} \qquad x' = \xi\alpha$$

$$E' = E + \frac{e^2\mathcal{E}^2}{m\omega^2} = \hbar\omega\left(n + \frac{1}{2}\right) \Rightarrow$$

$$\Rightarrow \boxed{E_n = \hbar\omega\left(n + \frac{1}{2}\right) - \frac{e^2\mathcal{E}^2}{2m\omega^2}}$$

№20



$$V(x)=\begin{cases}0, & 0<x<a\\ \infty, & x<0,\ x>a\end{cases}$$

$0<x<a$

$$\frac{\hbar^2}{2m}\Delta\psi+[E-V(x)]\psi=0 \qquad \text{р-ие Ш.}$$

II

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+E\psi=0$$

$$\frac{\partial^2}{\partial x^2}\psi+\frac{E2m}{\hbar^2}\psi=0$$

$$\varepsilon=\frac{E2m}{\hbar^2}$$

$$\begin{cases}\frac{1}{\varepsilon}\psi''+\psi=0\\ \psi(0)=0\\ \psi(a)=0\end{cases}$$

$$\psi''+\varepsilon\psi=0$$

$$\psi(x)=A\cos\sqrt{\varepsilon}x+B\sin\sqrt{\varepsilon}x$$

$$\psi(0)=0\Rightarrow A=0$$

$$\psi(a)=0\Rightarrow B\sin\sqrt{\varepsilon}a=0$$

$$\sin\sqrt{\varepsilon}a=0$$

$$\sqrt{\varepsilon_n}=\frac{\pi n}{a},\quad n=1,2,\dots$$

$$\varepsilon_n=\frac{E_n 2m}{\hbar^2}\Rightarrow E_n=\left(\frac{n\pi\hbar}{a}\right)^2\frac{1}{2m}, \text{ дискр. спектр.}$$

$\psi_n(x) = B\sin\frac{\pi n x}{a}$, $n = 1, 2, \ldots$ хв. ф-ї

соотв. B знайдем з умови

$$\int_{-\infty}^{+\infty} |\psi_n|^2 dx = |B|^2 \int_{-\infty}^{+\infty} \sin^2 \frac{\pi n x}{a}$$

$$|B|^2 \int_{-\infty}^{+\infty} \sin^2 kx \, dx = |B|^2 \int_{-\infty}^{+\infty} \left(\frac{1}{2} - \cos 2kx\right) dx =$$

$$= |B|^2 \frac{1x}{2}\Big|_0^a - \frac{\pi n}{a} \underbrace{\sin 2kx}_{=0}\Big|_0^a = |B|^2 \frac{a}{2} = 1 \Rightarrow$$

$$= |B|^2 = \frac{2}{a}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\frac{\pi n x}{a}$$

На відрізку $x < 0, x > a$, тобто вил.

I, III руху немає $\Rightarrow \psi_n = 0, E_n = 0$

B - дь

$$E_n = \left(\frac{\pi n \hbar}{a}\right)^2 \frac{1}{2m}$$

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin\frac{\pi n x}{a}, & 0 < x < a \\ 0 & x < 0, x > a \end{cases}$$

№21.

Довести $e^A e^B = e^{\frac{1}{2}[A,B]} e^{A+B}$

$$\left.\begin{matrix}[A,[A,B]]=0\\ [B,[A,B]]=0\end{matrix}\right\} \Rightarrow [A,B]=c=const$$

Введемо параметр x (дійсний)

$e^{xA}e^{xB} = \hat{W}(x)$ $e^{\hat{A}} = \sum_{n=0}^{\infty} \frac{\hat{A}^n}{n!}$

$\frac{d\hat{W}}{dx} = A e^{xA} e^{xB} + e^{x\hat{A}} B e^{xB}$

$[\hat{A}, e^{x\hat{B}}] = [\hat{A}, \sum_{n=0}^{\infty} \frac{(xB)^n}{n!}] = \sum_{n=0}^{\infty} \frac{x^n}{n!}[\hat{A}, \hat{B}^n] \ominus$

$[\hat{A}, \hat{B}^n] = AB^n - B^nA = AB \cdot B^{n-1} - B^nA = [AB = BA + C] =$

$= BAB^{n-1} + CB^{n-1} - B^nA = \ldots = nCB^{n-1}$

$\ominus \sum_{n=0}^{\infty} \frac{x^n}{n!} nCB^{n-1} = Cx \sum_{n=0}^{\infty} \frac{(xB)^{n-1}}{(n-1)!} = cx e^{xB}$

$Ae^{xB} - e^{xB}A = Cx e^{xB} \Rightarrow Ae^{xB} = e^{xB}A + Cx e^{xB}$

$\frac{dW}{dx} = e^{xA}[e^{xB}A + Cx e^{xB}] + e^{xA}e^{xB}B =$

$= e^{xA}e^{xB}[A+B] + Cx e^{xA}e^{xB} = [A+B+Cx]W$

$\frac{dW}{W} = [A+B+Cx]dx$

$W = C_1 e^{(A+B)x + \frac{Cx^2}{2}}$

$W(0) = 1 = C_1$

$W(x=1) = e^A e^B = e^{A+B} e^{[A,B]\frac{1}{2}}$

№22

$[\hat{L}\times\hat{L}] = i\hbar\hat{L}$, $\hat{L}$ - оп. м. к-сті. руху.

$xy = yx \quad p_x p_y = p_y p_z \quad p_x y - y p_x = -i\hbar\delta_{xy}$

$[L_i; L_j] = i\hbar L_k \varepsilon_{ijk}$

$$[L, L] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ L_x & L_y & L_z \\ L_x & L_y & L_z \end{vmatrix} =$$

$= i(L_y L_z - L_z L_y) - j(L_x L_z - L_z L_x) +$

$+ \vec{k}(L_x L_y - L_y L_x) = i\hbar(\vec{i}L_x + \vec{j}L_y + \vec{k}L_z) =$

$= i\hbar\hat{L}$

$L_y L_z - L_z L_y = (zp_x - xp_z)(xp_y - \cancel{yp_x}) -$

$-(xp_y - p_x y)(zp_x - p_z x) =$

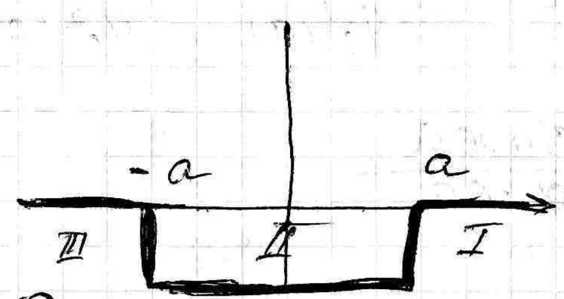
$= zp_x xp_y - \cancel{zp_x p_x y} - \cancel{p_z x xp_y} + p_z xp_x y -$

$- xp_y zp_x + \cancel{xp_y p_z x} + \cancel{p_x y zp_x} - p_x y p_z x =$

$= (zp_y - yp_z)(p_x x - xp_x) = i\hbar L_x$

$$V(x)=\begin{cases}-U_0 & -a<x<a\\ 0 & x<-a,\ x>a\end{cases}$$

№23



(I) симетричні розв'язки.

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2}\psi_s^I+\underbrace{U(x)}_{=0}=E\psi_s^I \qquad \Big|\ \frac{2\mu}{\hbar^2}$$

$$-\frac{\hbar^2}{2\mu}\frac{d^2\psi_s^I}{dx^2}=E\psi_s^I$$

$$\psi_s^{I}-k^2\psi_s^I=0 \qquad k=\sqrt{\frac{2\mu|E|}{\hbar^2}}$$

$$\psi_s^I=c_1e^{kx}+c_2e^{-kx}$$

$$c_1=0 \qquad \underline{\psi_s^I=c_2e^{-kx}}$$

(II) $$-\frac{\hbar^2}{2\mu}\frac{d}{dx^2}\psi_s^{II}-U_0\psi_s^{II}=E\psi_s^{II}$$

$$\psi_s''^{II}+æ^2\psi_s^{II}=0 \qquad æ^2=\frac{2\mu(U_0-E)}{\hbar^2}$$

$\psi_s''=A\cos æx+B\sin æx$, оскільки розв. симетричні то $B=0$

$$\psi_s^{II}=A\cos æx$$

(III) $$\psi_s^{III}=c_2e^{kx}$$

$$\psi_s(x)=\begin{cases}c_2e^{-kx}, & x>a & I\\ A\cos æx & |x|<a & II\\ c_2e^{kx} & x<-a & III\end{cases}$$

Умова неперервності хв. ф-ї

$$\frac{\psi_s^{I}(a)}{\psi_s'^{I}(a)} = \frac{\psi_s^{II}(a)}{\psi_s'^{II}(a)} \Leftrightarrow \frac{\psi_s^{I\,\prime}(a)}{\psi_s^{I}(a)} = \frac{\psi'^{II}(a)}{\psi_s^{II}(a)}$$

$$\frac{c_1(-\kappa)\,e^{-\kappa a}}{c_1\,e^{-\kappa a}} = A\,\frac{(-\varkappa)\sin\varkappa a}{A\cos\varkappa a}$$

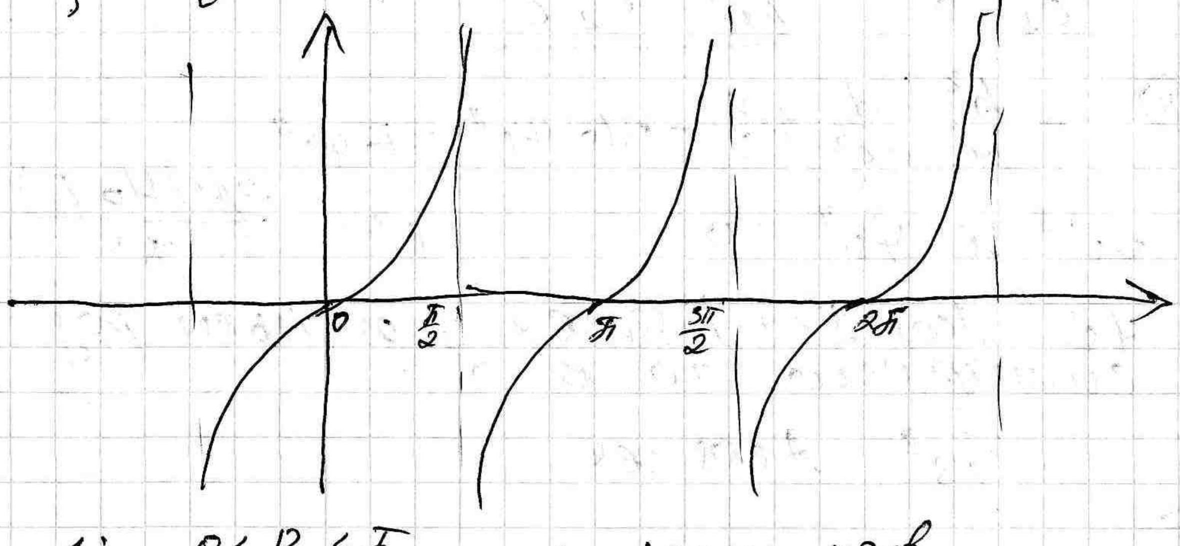
$$\kappa = \varkappa\,\mathrm{tg}\,\varkappa a \Rightarrow \kappa a = \varkappa a\,\mathrm{tg}\,a$$

$$\begin{cases} \varkappa a = \xi \\ \kappa a = \eta \end{cases} \qquad \begin{cases} \eta = \xi\,\mathrm{tg}\,\xi \\ \eta^2 + \xi^2 = R^2 \end{cases}$$

$$\eta = \sqrt{\frac{2m|E|}{\hbar^2}}\,a$$

$$\xi = \sqrt{\frac{2m|U_0 - E|}{\hbar^2}}\cdot a$$

$$\xi^2 + \eta^2 = \frac{2mU_0}{\hbar^2}a^2 = R^2 \qquad R^2 = \frac{2mU_0}{\hbar^2}a^2$$



1: $0 \le R \le \pi$: 1 сим. розв.

2: $\pi \le R \le 2\pi$: 2 сим. розв.

3: $2\pi < R < 3\pi$

$n_s\pi \le R \le (n_s+1)\pi$: 3 сим. розв.

Для антисимметричных

$$\psi_a = \begin{cases} C_1 e^{-\kappa x} & x > a \quad \mathrm{I} \\ B \sin \varkappa x & |x| < a \quad \mathrm{II} \\ -C_2 e^{\kappa x} & x < -a \quad \mathrm{III} \end{cases}$$

УН

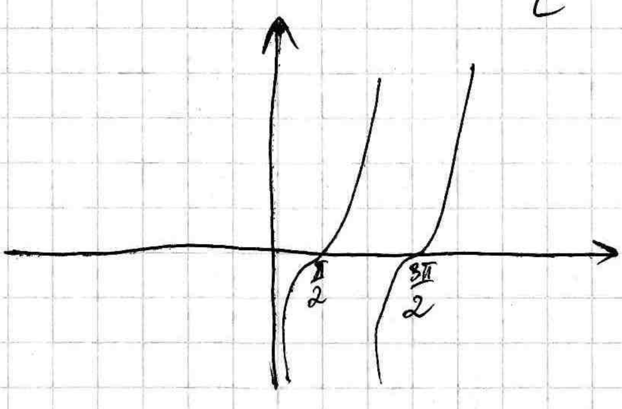
$$\frac{\dot{\psi}_a^{\mathrm{I}}}{\psi_a^{\mathrm{I}}}(a) = \frac{\dot{\psi}_a^{\mathrm{II}}}{\psi_a^{\mathrm{II}}}(a)$$

$$\frac{-C_1 \kappa e^{-\kappa a}}{C_1 e^{-\kappa a}} = \frac{B \varkappa \cos \varkappa x}{B \sin \varkappa x}$$

$$-\kappa = \varkappa \operatorname{ctg} \varkappa a$$

$$\kappa a = -\varkappa a \operatorname{ctg} \varkappa a$$

$$\begin{cases} \kappa a = \xi \\ \varkappa a = \eta \end{cases} \Rightarrow \begin{cases} \xi = -\eta \operatorname{ctg} \eta \\ \eta^2 + \xi^2 = R^2 \end{cases}$$



1: $\pi/2 \le R \le \frac{3\pi}{2}$

$\frac{3\pi}{2} \le R \le \frac{5\pi}{2}$

n_a: $(n_a - 1/2)\pi \le R \le (n_a + \frac{1}{2})\pi$

$$R^2 = \frac{2mU_0}{\hbar^2} a^2$$

Розрахувати для си-ми двох електронів вл. зн. і вл. вектори операторів №1.

$$\hat{\vec{S}}^2, \hat{S}_z, \vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$$

$$\Psi(\vec{r}, s, t) = \chi\,\varphi(\vec{r}, t)$$

$$\chi_1^+ = \begin{pmatrix}1\\0\end{pmatrix} \qquad \chi_1^- = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\chi_2^+ = \begin{pmatrix}1\\0\end{pmatrix} \qquad \chi_2^- = \begin{pmatrix}0\\1\end{pmatrix}$$

} вл. ф. $\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

Можливі варіації

$\chi_1^+\chi_1^+ \uparrow\uparrow$; $\chi_1^+\chi_2^- \uparrow\downarrow$; $\chi_1^-\chi_2^+ \downarrow\uparrow$; $\chi_1^-\chi_2^- \downarrow\downarrow$

$$S^2 = \frac{1}{4}\Big(\underbrace{\sigma_1^2}_{3I} + 2(\sigma_1\sigma_2) + \underbrace{\sigma_2^2}_{3I}\Big) = \Big[\sigma_i^2 = 3\underbrace{I}_{1.\text{ матриця}}, i = 1, 2\Big] =$$

$$= \frac{1}{2}\left(3I + \sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y} + \sigma_{1z}\sigma_{2z}\right)$$

$$\hat{S}_z = \frac{1}{2}(\sigma_{1z} + \sigma_{2z})$$

а) $S_z\chi_1^+\chi_2^+ = \frac{1}{2}(\sigma_{1z} + \sigma_{2z})\chi_1^+\chi_2^+ = \begin{bmatrix}\sigma_{iz}\chi^+ = \chi^+\\ \sigma_{iz}\chi^- = -\chi^-\end{bmatrix} =$

$$= \frac{1}{2}\left(\sigma_{1z}\chi_1^+\chi_2^+ + \sigma_{2z}\chi_1^+\chi_2^+\right) = \frac{1}{2}\left(\chi_1^+\chi_2^+ + \chi_1^+\chi_2^+\right) =$$

$= \chi_1^+\chi_2^+ \Rightarrow S_z = +1$.

$\chi_1^+\chi_2^+$ – вл. ф. оп. S_z

$$\begin{cases}\hat{S}_z\chi = \hbar\,\underbrace{\overset{1}{S_z}}_{\text{число}}\,\chi\\ \hat{S}^2\chi = \hbar^2 S(S+1)\end{cases}$$

б) $S_z\chi_1^+\chi_2^- = \frac{1}{2}(\sigma_{1z} + \sigma_{2z})\chi_1^+\chi_2^- = \frac{1}{2}(\sigma_{1z}\chi_1^+\chi_2^- +$

$+ \sigma_{2z}\chi_1^+\chi_2^-) = \frac{1}{2}(\chi_1^+\chi_2^- - \chi_1^+\chi_2^+) = 0 \Rightarrow \chi_1^+\chi_2^-$ – вл. ф-я S_z

в) $\vec{S}_z \chi_1^- \chi_2^+ = \frac{1}{2}(\sigma_{1z}\chi_1^-\chi_2^+ + \sigma_{2z}\chi_1^-\chi_2^+) =$
$= \frac{1}{2}(-\chi_1^-\chi_2^+ + \chi_1^-\chi_2^+) = 0$ — не вл. ф. $\vec{S}_z$ $\chi_1^-\chi_2^+$

г) $\vec{S}_z \chi_1^- \chi_2^- = \frac{1}{2}(\sigma_{1z}\chi_1^-\chi_2^- + \sigma_{2z}\chi_1^-\chi_2^-) =$
$= \frac{1}{2}(-\chi_1^-\chi_2^- - \chi_1^-\chi_2^-) = -1 \Rightarrow S_z = -1$
$\chi_1^-\chi_2^-$ — вл. ф. $\hat{S}_z$

Для $\vec{S}^2$:

а) $S^2\chi_1^+\chi_2^+ = \frac{1}{2}(3I\chi_1^+\chi_2^+ + \sigma_{1x}\chi_1^+\sigma_{2x}\chi_2^+ +$
$+\sigma_{1y}\chi_1^+\sigma_{2y}\chi_2^+ + \sigma_{1z}\chi_1^+\sigma_{2z}\chi_2^+) =$

$$= \left| \begin{array}{ll} \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; & \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \sigma_x\chi^+ = \chi^- & \sigma_x\chi^- = \chi^+ \\ \sigma_y\chi^+ = i\chi^- & \sigma_y\chi^- = -i\chi^+ \\ \sigma_z\chi^+ = \chi^+ & \sigma_z\chi^- = -\chi^- \end{array} \right| =$$

$= \frac{1}{2}(3\chi_1^+\chi_2^+ + \cancel{\chi_1^-\chi_2^-} + \cancel{i\chi_1^- \cdot i\chi_2^-} + \chi_1^+\chi_2^+) =$
$= \frac{1}{2}(4\chi_1^+\chi_2^+) \cancel{+\chi_1^-\chi_2^-(2+i)} = 2\chi_1^+\chi_2^+ \Rightarrow$
$\Rightarrow S(S+1) = 2 \Rightarrow$
$= S = 1$
$\chi_1^+\chi_2^+$ — вл. ф.

~~б) $S^2\chi_1^+\chi_2^- =$~~

в) $S^2\chi_1^-\chi_2^- = \frac{1}{2}(3\chi_1^-\chi_2^- + \sigma_{1x}\chi_1^-\sigma_{2x}\chi_2^- + \sigma_{1y}\chi_1^-\sigma_{2y}\chi_2^- +$
$+\sigma_{1z}\chi_1^-\sigma_{2z}\chi_2^-) = \frac{1}{2}(3\chi_1^-\chi_2^- + \chi_1^+\chi_2^+ + (-i\chi_1^+ \cdot -i\chi_2^+)$
$+(-\chi_1^-(-\chi_2^-))) = (\frac{1}{2}(3\chi_1^-\chi_2^- + \cancel{\chi_1^+\chi_2^+} - \cancel{\chi_1^+\chi_2^+} + \chi_1^-\chi_2^-)$
$= 2\chi_1^-\chi_2^- \Rightarrow S(S+1) = 2 \Rightarrow \underline{S=1}$, $\chi_1^-\chi_2^-$ — вл. ф.

б) $S^2\chi_1^+\chi_2^- = \frac{1}{2}(3I\chi_1^+\chi_2^- + \chi_1^-\chi_2^+ + i\chi_1^-(-i\chi_2^+) +$

$+ \chi_1^+ - \chi_2^-) = \frac{1}{2}(3\chi_1^+\chi_2^- + \chi_1^-\chi_2^+ + \chi_1^-\chi_2^+ -$

$- \chi_1^+\chi_2^-) = \frac{1}{2}(\cancel{2\chi_1^+\chi_2^-} = \chi_1^+\chi_2^- + \chi_1^-\chi_2^+$

г) $S^2\chi_1^-\chi_2^+ = \chi_1^-\chi_2^+ + \chi_1^+\chi_2^-$

χ-представлення в вигляді лін. комб.

$\chi = a\chi_1^+\chi_2^- + b\chi_1^-\chi_2^+$

$S^2\chi = (a+b)(\chi_1^+\chi_2^- + \chi_1^-\chi_2^+) = S(S+1)\chi$

$(a+b)(\chi_1^+\chi_2^- + \chi_1^-\chi_2^+) = S(S+1)(a\chi_1^+\chi_2^- + b\chi_1^-\chi_2^+)$

$a\chi_1^+\chi_2^- + a\chi_1^-\chi_2^+ + b\chi_1^+\chi_2^- + b\chi_1^-\chi_2^+ =$

$= S(S+1)a\chi_1^+\chi_2^- + S(S+1)b\chi_1^+\chi_2^-$

$$\begin{cases} a+b = S(S+1)a \\ a+b = S(S+1)b \end{cases}$$

$$\begin{cases} b = a(S(S+1)-1) \\ a = b(S(S+1)-1) \end{cases} \qquad \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$\begin{vmatrix} S(S+1)-1 & 1 \\ 1 & S(S+1)-1 \end{vmatrix} = 0 \Rightarrow$$

$\Rightarrow (S(S+1)-1)(S(S+1)-1) - 1 = 0$

$(S^2(S+1)^2 - 1)^2 = +1$

$S(S+1) - 1 = \pm 1$

$s(s+1)-1=1$ $\qquad$ $s(s+1)-1=-1$

$s(s+1)=2 \Rightarrow$ $\qquad$ $s(s+1)=0 \Rightarrow$

$\Rightarrow s=1$ $\qquad$ $\Rightarrow s=0$

$s=0: \; a=-b=\frac{1}{\sqrt{2}}$

$s=1: \; a=b=\frac{1}{\sqrt{2}}$

$^{s}\chi_{s_z}: \quad {}^{1}\chi_{1}=\chi_1^{+}\chi_2^{+}$

$\qquad {}^{1}\chi_{-1}=\chi_1^{-}\chi_2^{-}$

$${}^{0}\chi_{0}=a\chi_1^{+}\chi_2^{-}+b\chi_1^{-}\chi_2^{+}=\left[a=-b=\frac{1}{\sqrt{2}}\right]=$$
$$=\frac{1}{\sqrt{2}}\left(\chi_1^{+}\chi_2^{-}-\chi_1^{-}\chi_2^{+}\right)$$

$s_z=0;\ s=1$

$${}^{1}\chi_{0}=a\chi_1^{+}\chi_2^{-}+b\chi_1^{-}\chi_2^{+}=\left[a=b=\frac{1}{\sqrt{2}}\right]=$$
$$=\frac{1}{\sqrt{2}}\left(\chi_1^{+}\chi_2^{-}+\chi_1^{-}\chi_2^{+}\right)$$

№2

Розрахувати для n-го енерг. стану одн. осц. середні значення: $\langle x\rangle, \langle x^2\rangle, \langle p\rangle, \langle p^2\rangle$

$$\psi_n(\xi)=\frac{1}{\sqrt{2^n n!\sqrt{\pi}\alpha}}e^{-\xi^2/2}H_n(\xi), \quad \alpha=\sqrt{\frac{\hbar}{m\omega}}, \quad H_n=(-1)^n e^{\xi^2}\frac{d^n e^{-\xi^2}}{d\xi^n}$$

$$\langle x\rangle=\int\psi^*\psi x\,dx=\int_0^\infty|\psi|^2x\,dx$$

$$C_n^2=\sqrt{\frac{m\omega}{\pi\hbar}}\frac{1}{2^n n!}$$

$$\psi_n=C_n e^{-\xi^2/2}H_n(\xi), \qquad x=\alpha\xi$$

$$\langle x\rangle=\int_{-\infty}^{+\infty}C_n^2 x e^{-\xi^2}H_n^2(\xi)d\xi=$$

$$=\int_{-\infty}^{+\infty}C_n^2 x \cancel{e^{-\xi^2}} H_n (-1)^n \cancel{e^{\xi^2}}\frac{d^n e^{-\xi^2}}{d\xi^n}d\xi=$$

$$=\int_{-\infty}^{+\infty}C_n^2 x H_n (-1)^n\frac{d^n e^{-\xi^2}}{d\xi^n}d\xi=[x=\alpha\xi]=$$

$$=\int_{-\infty}^{+\infty}C_n^2\sqrt{\frac{\hbar}{m\omega}}\xi H_n(-1)^n\frac{d^n e^{-\xi^2}}{d\xi^n}d\xi=$$

$$=C_n^2\sqrt{\frac{\hbar}{m\omega}}(-1)^n\int_{-\infty}^{+\infty}\xi H_n(\xi)\frac{d^n e^{-\xi^2}}{d\xi^n}d\xi^n=$$

$$=C_n^2\sqrt{\frac{\hbar}{m\omega}}\int_{-\infty}^{+\infty}e^{-\xi^2}\frac{d^n(H_n(\xi))}{d\xi^n}d\xi=$$

$$=\left[\frac{d^n(H_n(\xi))}{d\xi^n}=\frac{d^n}{d\xi^n}\left(a_n\xi^{n+1}+a_{n-2}\xi^{n-1}+\dots\right)=a_n(n+1)!\xi,\right.$$

$$\left.a_n=2^n;\ a_{n-2}=-a_n\frac{n(n-1)}{4}\right]=$$

$$=C_n^2\sqrt{\frac{\hbar}{m\omega}}\int_{-\infty}^{+\infty}e^{-\xi^2}2^n(n+1)!\,\xi\,d\xi =$$

$$= C_n^2 \sqrt{\frac{\hbar}{m\omega}}\, 2^{n-1}(n+1)! = \sqrt{\frac{m\omega}{\pi\hbar}} \frac{1}{2^n n!} \frac{\hbar}{m\omega} 2^{n-1}(n+1)! =$$

$$= \sqrt{\frac{\hbar}{m\omega\pi}} \frac{n+1}{2}$$

через оператори нар. і зн.

$$\xi = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^+)$$

$\xi = \frac{x}{\alpha}$ – безр. величина

$$\langle n|\xi|n\rangle = \frac{1}{\sqrt{2}}\langle n(a|n\rangle + a^+|n\rangle) =$$

$$= \frac{1}{\sqrt{2}}(\langle n|n-1\rangle + \langle n|n+1\rangle) = |\langle n|m\rangle = \delta_{nm}| =$$

$$= 0.$$

$\langle x^2\rangle$ – теорема про віріал

$$\langle T\rangle = \langle U\rangle,\ \langle H\rangle = E = \langle T\rangle + \langle U\rangle = 2\langle U\rangle,$$

$$U = \frac{m\omega^2 x^2}{2}$$

$$E = m\omega^2\langle x^2\rangle = \hbar\omega(n+\tfrac{1}{2}) \Rightarrow$$

$$\Rightarrow \langle x^2\rangle = \frac{\hbar(n+\frac{1}{2})}{m\omega} \quad \frac{\hbar}{m\omega} = \alpha^2 \quad = \alpha^2(n+\tfrac{1}{2})$$

$$P_\xi = -\frac{i}{\sqrt{2}}(\hat{a} - \hat{a}^+)$$

$$\langle n|P_\xi|n\rangle = -\frac{i}{\sqrt{2}}(\langle n|n-1\rangle - \langle n|n+1\rangle) = 0$$

$$\langle p^2\rangle:\ \langle H\rangle = \frac{\langle p^2\rangle}{2m} + \frac{m\omega^2\langle x^2\rangle}{2} = \langle \hbar\omega(n+\tfrac{1}{2})\rangle$$

$$\Rightarrow \langle p^2\rangle = 2m\left(\langle\hbar\omega(n+\tfrac{1}{2})\rangle - \frac{m\omega^2\langle x^2\rangle}{2}\right) =$$

$$= 2m\left(\hbar\omega(n+\tfrac{1}{2}) - \frac{m\omega^2\hbar(n+\frac{1}{2})}{2m\omega}\right) \Rightarrow$$

$$\langle p^2 \rangle = \hbar \omega m \left(n + \frac{1}{2} \right)$$

№3

Розрахувати ефф. Штарка для атома водню, що знаходиться в пост. ел. полі в станах $n=1$, $n=2$.

$$\hat{H} = \hat{H}_0 + e\vec{\mathcal{E}}\vec{r} \qquad V_0 = e\vec{\mathcal{E}}z$$

$$= -e\mathcal{E}z$$

$$E_n = E_n^{(0)} + V_{nm} + \sum_{n\neq m} \frac{|V_{nm}|^2}{E_n - E_m}$$

для незбуреного стану : $\uparrow\downarrow \vec{e}$

$$\hat{H}_0 = -\frac{\hbar^2}{2\mu}\Delta - \frac{e^2}{r}$$

$$H_0 \psi_{nm}^{(0)} = E_n^{(0)} \psi_{nem}$$

$$l = \overline{0, n-1}, \quad m = \overline{-l, l}, \quad E_n^{(0)} = -\frac{R}{n^2}$$

для збурення

$n=1$

$$E_1 = E_1^{(0)} + \langle 100|V|100\rangle + \sum_{k\neq 1} \frac{|V_{1k}|^2}{E_1^{(0)} - E_k^{(0)}}$$

$$\hat{V} = -er\mathcal{E}\cos\theta$$

$$\psi_{100} - Ce^{-\frac{r}{a}}$$

$$V_{1n} = \int \psi_{100}^{*} \hat{V} \cdot \psi_{100}\, d\tau = \langle 100|V|100\rangle \Rightarrow$$

$$\Rightarrow V_{11} = 0$$

$$E_n^{(2)} = -\alpha\mathcal{E}^2$$

$n=2$: для вироджених станів $\alpha = 1,2,3,4$

$\psi_{200} = \psi_1$

$\psi_{010} = \psi_2$

$\psi_{21-1} = \psi_3$

$\psi_{211} = \psi_4$

$$\hat{H} = \underbrace{-\frac{\hbar^2}{2\mu}\Delta - \frac{e^2}{r}}_{\hat{H}_0} \underbrace{- er\mathcal{E}\cos\theta}_{\hat{V}}$$

$$\hat{H}_0 \psi_2 = E_2^{(0)} \psi_2$$

$$H_0 \psi_{nem} = E_n^{(0)} \psi_{nem}$$

$$(H_0 + V)\psi = E_2 \psi$$

$$\sum_{\alpha=1}^{4} E_2^{(0)} A_\alpha \psi_\alpha + V \sum_{\alpha=1}^{4} A_\alpha \psi_\alpha = E_2 \sum_{\alpha=1}^{4} A_\alpha \psi_\alpha$$

$$\sum_{\alpha=1}^{4} (E_2^{(0)} A_\alpha \delta_{\alpha\beta} + \sum_{\alpha=1}^{4} A_\alpha \int \psi_\beta^{-1} (er\mathcal{E}\cos\theta) \psi_\alpha d\tau = 0$$

$$\int \psi_{2em} V \psi_{2e'm'} d\tau = / \psi_{2em} = R_{2e}(r) Y_{em}(\theta, \varphi) =$$

$$= R_{2e}(r) C_{em} P_e^{(m)}(\cos\theta) e^{im\varphi} / =$$

$$= \int_0^\infty r^2 \cdot r R_{2e}(r) C_{em} C_{e'm'} R_{2e'}(r) \int_0^{\pi} \cos\theta \sin\theta \times$$

$$\times P_e^{(m)}(\cos\theta) P_{e'}^{(m)}(\cos\theta) \int_0^{2\pi} \underbrace{e^{-i(m-m')\varphi}}_{m=m'} d\varphi$$

Ненульовими будуть V_{21} і V_{12}, бо тільки для них:

$\alpha = 1, 2$, $m = m' = 0$.

Тоді можна записати:

$$(*)\begin{pmatrix} -\Delta E & V_{12} & 0 & 0 \\ V_{12} & -\Delta E & 0 & 0 \\ 0 & 0 & -\Delta E & 0 \\ 0 & 0 & 0 & -\Delta E \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Визначник $(\Delta E)^2\left[(\Delta E)^2 - (V_{12})^2\right] = 0$

$\Delta E = 0$ або $\Delta E = \pm V_{12}$

Якщо $\Delta E = 0$, то підст. в (*) нехай $A_0 = 0$,

$A_1 V_{12} = 0$ $\qquad$ $A_4 = 1 \Rightarrow \psi = \psi_3 = \psi_{21-1}$

$A_2 V_{12} = 0$

$A_3, A_4 - \forall$

Якщо $\Delta E = V_{12}$; $-V_{12}A_1 + V_{12}A_2 = 0 \Rightarrow A_1 = A_2$
$A_3 = A_4 = 0$

$\Delta E = -V_{12}$: $A_1 = -A_2$, $A_3 = A_1 = 0$

$\psi = A(\psi_{210} + \psi_{200}) = / \int \psi^* \psi \, d\tau = 1 \Rightarrow 2|A|^2 = 1 / =$

$= \frac{1}{\sqrt{2}}(\psi_{210} + \psi_{200})$

$\psi_{200} = \frac{1}{\sqrt{32\pi}}(2-\rho)e^{-\rho/2}$

$\psi_{210} = \frac{1}{\sqrt{32\pi}}\rho e^{-\frac{\rho}{2}}\cos\theta$

Рдрахуємо : V_{12}

$\int \psi^*_{200} \underset{-z e \mathcal{E} \cos\theta}{V} \psi_{210} \, d\tau = -e\mathcal{E}a \frac{1}{32\pi}\int_0^\infty \rho^4(2-\rho)e^{-\rho}d\rho \,\times$

$\times \int_0^\pi \cos^2\theta \sin\theta \, d\theta \int_0^{2\pi} d\varphi =$

$\left[\int_0^\pi \cos^2\theta \, d\cos\theta = -\frac{\cos^3\theta}{3}\Big|_0^\pi = \frac{2}{3} \right]$

$$\int_0^\infty 2\rho^4 e^{-\rho} d\rho - \int_0^\infty \rho^5 e^{-\rho} d\rho = \left[\int_0^\infty \rho^n e^{-\rho} d\rho = n! \right] =$$

$$= -3 \cdot 4!$$

$$\textcircled{=} \; 2\pi \cdot \frac{2}{3} \cdot 4! \; \frac{e\mathcal{E}a}{32\pi} = 3e\mathcal{E}a$$

№4.

Знайти співвідношення між ядрами $L(x,x')$ та $L(p,p')$ оператора $\hat{L}$ в x- та p-представленнях.

а) $L(x,x') = \langle x|L|x'\rangle$

б) $L(p,p') = \langle p|L|p'\rangle$

а) $$\langle x|L|x'\rangle = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \langle x|p\rangle\langle p|L|p'\rangle\langle p'|x'\rangle dp dp' =$$

$$= \left(\frac{1}{\sqrt{2\pi\hbar}}\right)^2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} L(p,p') e^{\frac{i}{\hbar}(px-p'x)}$$

$$\psi_p = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{i}{\hbar}p_0x} \qquad \psi_x = \frac{1}{\sqrt{2\pi\hbar}} e^{-\frac{i}{\hbar}px_0}$$

б) $$\langle p|L|p'\rangle = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \langle p|x\rangle\langle x|L|x'\rangle\langle x'|p'\rangle dx dx' =$$

$$= \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} L(x,x') e^{-\frac{i}{\hbar}(px-p'x')} dx dx'$$

Знайти x в. ф-ю. зар. час., що зн. в пост. ел. пол.

№25

$x_{коор.}$, p - імп.

$F_x = F = const$, $\qquad x \to \hat{x} = i\hbar \frac{\partial}{\partial p_x}$

$F_x = E_x e$. $\qquad p_x \to \hat{p}_x = -i\hbar \frac{\partial}{\partial x}$

$\vec{F_x} = -\vec{\nabla} U \Rightarrow U = -eEx = -F_x x$

$U = -eE i\hbar \frac{\partial}{\partial p_x}$

$T = \frac{p_x^2}{2\mu}$

$\hat{H}\psi = E\psi$

$\hat{H} = T - U$

$\frac{p_x^2}{2\mu}\psi - eE_0 i\hbar \frac{d\psi}{dp_x} = E\psi$

$\frac{d\psi}{dp_x} = \left(E - \frac{p_x^2}{2\mu}\right)\frac{i}{\hbar e E_0}$ ← інтегруємо

$\psi_E(p_x) = C e^{\frac{i}{\hbar e E_0}\left(Ep_x - \frac{p_x^3}{6\mu}\right)}$

$\ln\psi = \cancel{\frac{Ep}{i}} \frac{1}{i\hbar e E_*}\left\{\frac{p^3}{6\mu} - Ep\right\}$

$\int\limits_{-\infty}^{+\infty} \psi^*_{E(p_x)}\psi(p_x)\, dp_x = \delta(E - E')$

$\int\limits_{-\infty}^{+\infty} |C|^2 e^{\frac{-i}{\hbar e E_0}\left(E' p_x - \frac{p_x^3}{6\mu}\right)} e^{\frac{i}{\hbar e E_0}\left(E' p_x - \frac{p_x^3}{6\mu}\right)} dp_x =$

$= |C|^2 \underbrace{\int\limits_{-\infty}^{+\infty} e^{\frac{i}{\hbar e E_0}(E - E')p_x} dp_x}_{2\pi\delta(E-E')/\hbar e E_0} = |C|^2 2\pi\delta\left(\frac{E - E'}{\hbar e E_0}\right) \Rightarrow$

$$\Rightarrow |C|^2 \delta(E-E')\, 2\pi\hbar e E_0 = \delta(E-E') \Rightarrow$$

$$\Rightarrow C = \frac{1}{\sqrt{2\pi e\hbar E_0}}$$

$\psi(p_x) \to \psi(x)$

$\psi(x) = \int \psi(p)\, \psi_p(x)\, dp$

$=$

$$\psi(p) = Ce^{(\cdot\ \cdot\ \cdot\ \cdot)}$$

$\psi_E(x) = \int \psi_E(p)$

$$\psi(x) = \int_{-\infty}^{+\infty} \frac{1}{2\pi\hbar\sqrt{E_0}}\, e^{-ip_x x}\, e^{\frac{i}{\hbar e E_0}\left(Ep_x - \frac{p_x^3}{6m}\right)} dp_x$$

Розрахувати рівні енергії та хв. ф. част. №6 зі спіном $S=1$ (в одиницях $\hbar$), якщо $\hat{H}$ має вигляд:

$$\hat{H} = A\hat{S}_x^2 + B\hat{S}_y^2 + C\hat{S}_z^2$$

де A, B, C - довільні сталі,

$$\hat{S}_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{S}_y = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$[S^2, \hat{S}_z] = 0$$

$$S_z = \pm 1, 0$$

$$|+1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad |0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad |-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\hat{S}^2 \chi = S(S+1)\chi$$

$$S^2 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{cases} S_+ = S_x + iS_y \\ S_- = S_x - iS_y \end{cases} \Rightarrow \quad S_x = \frac{1}{2}(S_+ + S_-), \quad S_y = \frac{1}{2i}(S_+ - S_-)$$

$$[S_x, S_y] = i\overset{1}{\hbar} S_z$$

$$[S_+, S_-] = 2S_z$$

$$[S_+, S_-] = S_+S_- - S_-S_+ = (S_x + iS_y)(S_x - iS_y) - (S_x - iS_y)(S_x + iS_y) = 2S_z$$

$$S_x^2 = \frac{1}{4}(S_+^2 + S_-^2 + S_+S_- + S_-S_+)$$

$$S_y^2 = -\frac{1}{4}(S_+^2 + S_-^2 - S_+S_- - S_-S_+)$$

$$S_x^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}; \quad S_y^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}; \quad S_z^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\hat{H} = \frac{A}{\cancel{2}}\left(S_x^2 + \frac{B}{\cancel{2}} S_y^2 + C S_z^2\right.$$

$$\hat{H} = \frac{1}{2}(A-B)\left(S_+^2 + S_-^2\right) + \frac{1}{2}(A+B)\left(S_+S_- + S_-S_+\right) + C S_z^2$$

$$\hat{H} = \begin{pmatrix} \frac{A}{2} + \frac{B}{2} + C - E & 0 & \frac{A}{2} - \frac{B}{2} \\ 0 & A+B-E & 0 \\ \frac{A}{2} - \frac{B}{2} & 0 & \frac{A}{2} + \frac{B}{2} + C - E \end{pmatrix}$$

$$\det \hat{H} = 0$$

$$(A+B-E)\left(\frac{A}{2} + \frac{B}{2} + C - E\right)\left(\frac{A}{2} + \frac{B}{2} + C - E\right) -$$

$$-(A+B-E)\left(\frac{A}{2} - \frac{B}{2}\right)\left(\frac{A}{2} - \frac{B}{2}\right) = 0$$

$$(A+B-E)\left[\left(\frac{A}{2} + \frac{B}{2} + C - E\right)^2 - \left(\frac{A}{2} - \frac{B}{2}\right)^2\right] = 0$$

$$E_1 = A+B$$

$$\frac{A}{2} + \frac{B}{2} + C - E = \frac{A}{2} - \frac{B}{2}$$

$$E_2 = B + C$$

$$\frac{A}{2} + \frac{B}{2} + C - E = -\frac{A}{2} + \frac{B}{2}$$

$$E_3 = A + C$$

1) $E_1 = A + B$

$$H\chi = E\lambda$$

$$H\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \left(C - \frac{A}{2} - \frac{B}{2}\right) a + \left(\frac{A}{2} - \frac{B}{2}\right) c = 0 \\ \left(C - \frac{A}{2} - \frac{B}{2}\right) c + \left(\frac{A}{2} - \frac{B}{2}\right) a = 0 \end{cases} \Rightarrow$$ р-ня несумісні a, b — ?

$$\chi_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ - вл. вектор

2) $E_2 = B + C$

$$\left(\frac{A}{2} - \frac{3B}{2}\right) a + \left(\frac{A}{2} - \frac{B}{2}\right) c = 0$$

$$(C + A) b = 0$$

$$\left(\frac{A}{2} - \frac{B}{2}\right) a + \left(\frac{A}{2} - \frac{B}{2}\right) c = 0$$

$$a = c = \frac{1}{\sqrt{2}}, \quad b = 0 \Rightarrow \chi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

3) $E_3 = A + C$

$$\chi_3 = \sqrt{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

№27

$$V(x) = \frac{1}{2} m\omega^2 x^2 + \alpha x^3$$

$$\underbrace{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2} m\omega^2 x^2}_{=\hat{H}_0} + \alpha x^3 = \hat{H}$$

$$\hat{H}_0 \psi_n^{(0)} = E_n^{(0)} \psi_n^{(0)}$$

$$E_n^{(0)} = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$E_n = E_n^{(0)} + W_{nm} + \sum \frac{|W_{nm}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$W_{mn} = \int_{-\infty}^{+\infty} \psi_n^* W \psi_m^{(0)} dx = \langle n | W | m \rangle$$

$$\langle n | x^3 | n \rangle = \langle n | x^2 x | n \rangle = \sum_n \langle n | x^2 | k \rangle \langle k | x | n \rangle$$

$$\alpha = \sqrt{\frac{\hbar}{\pi\omega}}, \quad x = \frac{\alpha}{\sqrt{2}}\left(\hat{a} + \hat{a}^+\right) \qquad (1)$$

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1 \qquad\qquad \hat{a}^+\hat{a}|n\rangle = n\,|n\rangle$$

$$\hat{a}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle \qquad\qquad \langle n|m\rangle = \delta_{nm}$$

$$\langle k | x | n \rangle = \frac{\alpha}{\sqrt{2}}$$

$$\hat{x}^2 = \frac{\alpha^2}{2}\left(\hat{a} + \hat{a}^+\right)\left(\hat{a} + \hat{a}^+\right) = \frac{\alpha^2}{\alpha}\left(\hat{a}\hat{a} + \hat{a}^+\hat{a} + \right.$$

$$\left. + \hat{a}\hat{a}^+ + \hat{a}^{+2}\right) = \frac{\alpha^2}{\alpha}\left(\hat{a}^2 + 2n + 1 + \hat{a}^{+2}\right)$$

$$\langle n | x^2 | k \rangle =$$

$$\hat{a}^2|k\rangle = \hat{a}\hat{a}\,|k\rangle = \sqrt{k}\,\hat{a}\,|k-1\rangle = \sqrt{k^2-1}\,|k-2\rangle$$

$$\hat{a}^{+2}|k\rangle = \sqrt{k+1}\,\hat{a}^+\,|k+1\rangle = \sqrt{(k+1)(k+2)}\,|k+2\rangle$$

$$= \frac{\alpha^2}{2}\langle n | \hat{a}^2 + 2\hat{n} + 1 + \hat{a}^{+2} | k \rangle = \left[2\hat{n}+1|k = 2k+1|k\rangle\right].$$

$$= \frac{\alpha^2}{\alpha}\left(\sqrt{k(k-1)}\,\delta_{n,k-2} + (2k+1)\,\delta_{n,k} + \sqrt{(k+1)(k+2)}\,\delta_{n,k+2}\right]$$

$$\langle n|x^2|n\rangle = \sum_{k=n\pm1}\left(\sqrt{k(k-1)}\,\delta_{n,k-2} + (2k+1)\delta_{n,k} +\right.$$
$$\left.+\sqrt{(k+1)(k+2)}\,\delta_{n,k+2}\right) + \ldots =$$
$$= \langle n|x^2|n-1\rangle\langle n-1|x|n\rangle +$$
$$+ \langle n|x^2|n+1\rangle\langle n+1|x|n\rangle = 0$$
$$W^1_{nn} = 0$$

Другу попр. ен. ја фо-ло

$$\sum_{m\neq n}\frac{|W_{nm}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\langle n|x^3|m\rangle = \langle n|x^2|k\rangle\underbrace{\langle k|x|m\rangle}_{k=m\pm1} =$$
$$= \langle n|x^2|m-1\rangle\langle m-1|x|m\rangle +$$
$$+\langle n|x^2|m+1\rangle\langle m+1|x|m\rangle =$$

$$\left[\begin{array}{l}\langle m-1|x|m\rangle = \frac{\alpha}{\sqrt2}\sqrt{m}\\ \langle m+1|x|m\rangle = \frac{\alpha}{\sqrt2}\sqrt{m+1}\end{array}\right]$$

$$= \langle n|x^2|m-1\rangle\frac{\alpha}{\sqrt2}\sqrt{m} + \langle n|x^2|m+1\rangle\frac{\alpha}{\sqrt2}\sqrt{m+1} =$$
$$= \left(\frac{\alpha}{\sqrt2}\right)^3\left[\sqrt{(m-1)(m-2)}\,\delta_{n,m-2} + \sqrt{m}\,(2m-1)\,\delta_{n,m-1} +\right.$$
$$+ \sqrt{m^2(m+1)}\,\delta_{n,m+1} + \sqrt{(m+1)^2}\,m\,\delta_{n,m-1} +$$
$$\left.+\sqrt{m+1}\,(2m+3)\,\delta_{n,m+1} + \sqrt{(m+1)(m+2)(m+3)}\,\delta_{n,m+3}\right] =$$
$$= \left(\frac{\alpha}{\sqrt2}\right)^3\left\{\sqrt{m(m-1)(m-2)}\,\delta_{n,m-3} + 3m^{3/2}\,\delta_{n,m-1} +\right.$$
$$\left.3(m+1)^{3/2}\,\delta_{n,m+1} + \sqrt{(m+1)(m+2)(m+3)}\,\delta_{n,m+3}\right\}$$

$$\sum_{m\neq n}\frac{|W_{nm}|^2}{E_n^{(0)}-E_m^{(0)}}=\frac{|W_{n,n-1}|^2}{E_n^{(0)}-E_{n-1}^{(0)}}+\frac{|W_{n,n+1}|^2}{E_n^{(0)}-E_{n+1}^{(0)}}+$$

$$+\frac{|W_{n,n-3}|^2}{E_n^{(0)}-E_{n-3}^{(0)}}+\frac{|W_{n,n+3}|}{E_n^{(0)}-E_{n+3}^{(0)}}=\frac{1}{\hbar\omega}\Big(|W_{n,n-1}|^2-$$

$$-|W_{n,n+1}|^2+\frac{1}{3}|W_{n,n-3}|^2-\frac{1}{3}|W_{n,n+3}|^2=$$

$$=\frac{\alpha^2\cdot\left(\sqrt{\frac{\hbar}{m\omega}}\right)^6}{8\hbar\omega}\Big(\left(3n^{3/2}\right)^2-\left(3(n+1)^{3/2}\right)^2+$$

$$+\frac{1}{3}\left(\sqrt{n(n-1)(n-2)}\right)^2-\frac{1}{3}\left(\sqrt{(n+3)(n+2)(n+1)}\right)^2\Big)=$$

$$=\frac{\alpha^2\left(\sqrt{\frac{\hbar}{m\omega}}\right)^6}{24\hbar\omega}\Big\{27\left(n^3-(n+1)^3\right)+\big(n(n-1)(n-2)-$$

$$-(n+1)(n+2)(n+3)\big)$$

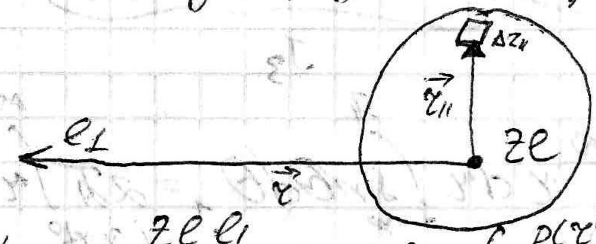
В-ды : $-\dfrac{\alpha^2\left(\frac{\hbar}{m\omega}\right)^3}{\hbar\omega}\left\{30n^2+30n+11\right\}$

№8

$\rho(r) = -e\rho_0 \exp(-\frac{r}{a})$;

$\int \rho(r)\,dr = -Ze$

диф. переріз пружного розсіяння?



$$V(r') = \frac{Zee_1}{r'} - ee_1 \int \frac{\rho(r'')\,d\vec{r}''}{|\vec{r}' - \vec{r}''|}$$

$\rho = \rho_0 e^{-r/a}$ $\qquad q = 2\sin\frac{2\alpha}{2}$

$$f(\alpha) = -\frac{2m}{\hbar^2 q}\int_0^\infty r V(r)\sin qr\,dr =$$

$$\rho_0 \underbrace{\int_0^\infty dr\, r^2 e^{-r/a}}_{J_1} \underbrace{\int_0^\pi \sin\Theta\, d\Theta \int_0^{2\pi} d\varphi}_{4\pi} = Z$$

$$\left[J_1 = \int_0^\infty r^2 e^{-r/a}\,dr = a^3 \int_0^\infty e^{-\left(\frac{r}{a}\right)} d\left(\frac{r}{a}\right) = 2a^3 \Rightarrow \right.$$

$$\left. \Rightarrow 8\pi a^3 \rho_0 = Z \Rightarrow \rho_0 = \frac{Z}{8\pi a^3} \right]$$

$$f(\alpha) = -\frac{2m}{\hbar^2 q}\int_0^\infty r\, \sin qr \left(\frac{Zee_1}{r} - \frac{ee_1 Z}{8\pi a^3} \iiint d\vec{r}'' \cdot \frac{e^{-r''/a}}{|r - r''|} \right) dr \ominus$$

$$\left[\int_0^\infty \sin qr\,dr = \frac{1}{q} \right]$$

$$J_2 = \int d\vec{r}\, e^{i\vec{q}\vec{r}} \int d\vec{r}'' \frac{e^{-r''/a}}{|\vec{r} - \vec{r}''|} = \left[\vec{r}'' = \vec{r} - \vec{r}' \right] =$$

$$= \int d\vec{r}\, e^{i\vec{q}\vec{r}} \int (-d\vec{r}')\, e^{-(r''/a)} \ominus$$

$$\circledS \int d\vec{r}''\, e^{-r''/a} \int d\vec{r}'\, e^{i\vec{q}(\vec{r}'+\vec{r}'')} =$$

$$= \underbrace{\int d\vec{r}''\, e^{-\frac{r''}{a}+i\vec{q}\vec{r}''}}_{J_4} \underbrace{\int d\vec{r}\, \frac{e^{i\vec{q}\vec{r}'}}{r'}}_{J_3} =$$

$$J_3 = 2\pi \int e^{iqr\cos\theta}\, r\,dr \int_0^{\pi} \sin\theta\, d\theta = 2\pi \int_0^{\infty} r^2 dr \cdot$$

$$\cdot \int_0^{\pi} e^{iqr\cos\theta}(-d\cos\theta) = 2\pi \int_0^{\infty} r\,dr \int_{-1}^{1} e^{iqrt}\,dt =$$

$$= 2\pi \int_0^{\infty} r\,dr\, \frac{1}{iqr}\, e^{iqrt}\Big|_{-1}^{1} = \frac{2\pi}{iq}\int_0^{\infty} dr\left(e^{iqr}-e^{-iqr}\right) =$$

$$= \frac{4\pi}{q^2}$$

$$J_4 = \int_0^{\infty} d\vec{r}\, e^{-\frac{r}{a}+i\vec{q}\vec{r}} = \left[\int_0^{\infty} e^{-\alpha t} t^n dt = \frac{n!}{\alpha^{n+1}}\right] =$$

$$= \frac{2\pi}{iq}\int_0^{\infty} dr\, \frac{r^2}{r}\left[e^{-\frac{r}{a}+iqr} - e^{-\frac{r}{a}-iqr}\right] =$$

$$= \frac{2\pi}{iq}\left(\frac{1}{(\frac{1}{a}-iq)^2} - \frac{1}{(\frac{1}{a}+iq)^2}\right) = \frac{2\pi}{iq}\, \frac{\frac{i4q}{a}}{((\frac{1}{a})^2+q^2)^2} =$$

$$= \frac{8\pi}{a}\, \frac{1}{((\frac{1}{a})^2+q^2)^2}$$

$$f(\alpha) = \frac{2m}{q^2\hbar^2}\left(Zee_1 - \frac{e_1 e Z}{8\pi a^3}\cdot\frac{8\pi}{a}\left(\frac{1}{((\frac{1}{a})^2+q^2)^2}\right)\right)^2 =$$

$$= -\frac{2m\, Zee_1}{\hbar^2 q^2}\left(1 - \frac{1}{(1+q^2a^2)^2}\right)$$

$$\sigma = |f(\alpha)|^2$$

Довести, що не $\exists$ жодного ненульового вл. стану для оператора народження a^+ №9

$$a^+ = \frac{1}{\sqrt{2}}\left(\xi - \frac{d}{d\xi}\right), \quad \xi = \frac{x}{\alpha} \quad \alpha = \sqrt{\frac{\hbar}{m\omega}}$$

$$a^+\psi(\xi) = \lambda\psi(\xi)$$

$$\frac{1}{\sqrt{2}}\left(\xi\psi - \frac{d\psi}{d\xi}\right) = +\lambda\psi$$

$$\frac{d\psi}{d\xi} + (\xi + \sqrt{2}\lambda)\psi = 0$$

$$\frac{d\psi}{\psi} = (\xi - \sqrt{2}\lambda)d\xi$$

$$\psi = C e^{-\xi^2/2 - \sqrt{2}\lambda\xi}$$

При $\forall\lambda$ не існує обм. вл. ф.

Знайти прямим варіаційним методом ен-ію основного стану атома водню, вибравши хв. ф-ію у вигляді **Б№10**

$$\psi = A(1+\alpha r)e^{-\alpha r}$$

$$U = -\frac{e^2}{r} \;; \quad E_n = \frac{-Ry}{n^2}$$

(1) $\int |\psi(\alpha,\beta,\ldots)|^2 d\tau = 1$

$$\int |A(1+\alpha r)e^{-\alpha r}| d\tau = \left[d\tau = r^2 \sin\theta\, d\theta\, d\varphi\, dr\right] =$$

$$= A^2 \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta \int_0^\infty r^2(1+\alpha r)^2 e^{-2\alpha r}\, dr =$$

$$= |A|^2 4\pi \int_0^\infty r^2(1+\alpha r)^2 e^{-2\alpha r}\, dr = \left[\begin{matrix} 2\alpha r = x \\ dx = 2\alpha\, dr \end{matrix}\right] =$$

$$= |A|^2 4\pi \int_0^\infty e^{-x}\left(1+\frac{x}{2}\right)^2 \frac{x^2}{(2\alpha)^2}\frac{1}{2\alpha}\, dx =$$

$$= 4\pi|A|^2 \int_0^\infty e^{-x}\left(1+x+\frac{x^2}{4}\right)\frac{x^2}{(2\alpha)^3}\, dx =$$

$$= \frac{4\pi|A|}{(2\alpha)^3}\int_0^\infty e^{-x}\left(x^2+x^3+\frac{x^4}{4}\right) dx =$$

$$= \left[\int_0^\infty e^{-x}x^2\, dx = 2 \;; \int_0^\infty e^{-x}x^3\, dx = 6 \;; \frac{1}{4}\int_0^\infty e^{-x}x^4 = 6\right] =$$

$$= \frac{7\pi|A|^2}{\alpha^3} = 1 \Rightarrow |A|^2 = \frac{\alpha^3}{7\pi}, \; A = \sqrt{\frac{\alpha^3}{7\pi}}$$

(2) $T = \frac{\hbar^2}{2m}\int \left|\frac{d\psi}{dr}\right|^2 d\tau = \left[\frac{d\psi}{dr} = \left(Ae^{-\alpha r} + A\alpha r e^{-\alpha r}\right)' = \left(\cancel{-A\alpha e^{-\alpha r}} + \cancel{A\alpha e^{-\alpha r}} - A\alpha^2 r e^{-\alpha r}\right)\right] =$

$$= \frac{4\pi\hbar^2}{2m}\int_0^\infty A^2\alpha^4 r^4 e^{-2\alpha r}\, dr = \frac{2\pi\hbar^2}{m}\int_0^\infty A^2\alpha^4 r^4 e^{-2\alpha r}\, dr =$$

$$= \frac{2\pi A^2\hbar^2\alpha^4}{m}\int_0^\infty r^4 e^{-2\alpha r}\, dr = \left[\int_0^\infty r^4 e^{-2\alpha r}\, dr = \frac{3}{4\alpha^5}\right] =$$

$$= \frac{2\pi\,\hbar^2\alpha^4}{m}\,\frac{\alpha^3}{7\pi}\cdot\frac{3}{4\alpha^5} = \frac{3}{14}\,\frac{\hbar^2\alpha^2}{m}$$

$$\bar{U} = \int |\psi|^2\, U\, dr = 4\pi\int -\frac{e^2}{r}\,r^2\left(A^2 e^{-2\alpha r} + 2A^2\alpha r e^{-2\alpha r} + \right.$$

$$\left. + A^2\alpha^2 r^2 e^{-2\alpha r}\right) dr = 4\pi(-e^2)\,A^2 \underbrace{\int_0^\infty e^{-2\alpha r}\, r\, dr}_{\frac{1}{(2\alpha)^2}} +$$

$$+ \underbrace{2\int_0^\infty \alpha r^2 e^{-2\alpha r}\, dr}_{2\alpha\,\frac{2!}{(2\alpha)^3}} + \underbrace{\alpha^2\int_0^\infty r^3 e^{-2\alpha r}\, dr}_{\frac{3!}{(2\alpha)^4}} =$$

$$=$$

$$= \pi(-e)^2\,\frac{\alpha^3}{7\pi}\,\frac{9}{2\alpha^2} =$$

$$= -\frac{9}{14}\,e^2\alpha$$

$$\hat{H} = \bar{T} + \bar{U}$$

$$\bar{H} = \frac{3}{14}\,\frac{\hbar^2\alpha^2}{m} - \frac{9}{14}\,e^2\alpha$$

$$\frac{\partial \bar{H}}{\partial \alpha} = \frac{3}{14}\,\frac{\hbar^2}{m}\,2\alpha_0 - \frac{9}{14}\,e^2 \Rightarrow \alpha_0 = \frac{3}{2}\,\frac{me^2}{\hbar^2}$$

$$E_1 = \frac{3}{14}\,\frac{\hbar^2}{m}\left(\frac{9}{4}\,\frac{m^2e^4}{\hbar^4}\right) - \frac{9}{14}\,e^2\left(\frac{3}{2}\,\frac{me^2}{\hbar^2}\right) =$$

$$= \frac{27}{56}\,\frac{me^4}{\hbar^2} - \frac{27}{28}\,\frac{e^4m}{\hbar^2} = \frac{27-54}{56}\,\frac{e^4m}{\hbar^2} =$$

$$= -\frac{27}{56}\,\frac{e^4m}{\hbar^2} = -\frac{27}{28}\,Ry$$

$$Ry = \frac{e^4m}{2\hbar^2}$$

Знайти вл. зн. і вл. вектори операторів Паулі №11
$\sigma_x \; \sigma_y \; \sigma_z$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x \chi = \lambda \chi \qquad \chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix} \qquad \begin{cases} a = \lambda b \\ b = \lambda a \end{cases} \quad \begin{cases} \lambda a - b = 0 \\ -a + \lambda b = 0 \end{cases}$$

$$\begin{vmatrix} \lambda & -1 \\ -1 & \lambda \end{vmatrix} = \lambda^2 - 1 = 0 \Rightarrow \begin{matrix} \lambda_1 = 1 \\ \lambda_2 = -1 \end{matrix}$$

$$a = b = \frac{1}{\sqrt{2}} \qquad \text{з умов нормування} \qquad \chi^* \psi = a^* a + b^* b = |a|^2 + |b|^2 = 1$$
$$a = -b = \frac{1}{\sqrt{2}}$$

$$\chi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \chi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ +1 \end{pmatrix}$$

$$\sigma_y: \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{cases} -ib = \lambda a \\ ia = \lambda b \end{cases} \qquad \cancel{\begin{cases} -ib - \lambda a = 0 \\ ia - \lambda b = 0 \end{cases}}$$

$$\lambda a + ib = 0$$
$$-ia + \lambda b = 0$$

$$\begin{vmatrix} \lambda & i \\ -i & \lambda \end{vmatrix} = \lambda^2 - 1 \Rightarrow \lambda = \pm 1$$

$$\lambda = 1 \qquad a = -ib \qquad \chi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$$

$$\lambda = -1 \qquad a = ib \qquad \chi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$$

σ_z : $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix}$

$$a = \lambda a$$
$$-b = \lambda b$$

$$\begin{cases} \lambda a - a = 0 \\ \lambda b + b = 0 \end{cases} \Rightarrow \begin{cases} a(\lambda - 1) = 0 \\ b(\lambda + 1) = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{vmatrix} \lambda - 1 & 0 \\ 0 & \lambda + 1 \end{vmatrix} = 0$$

$\lambda_1 = 1$: $a \; \forall, \; b = 0 \Rightarrow \chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad a = 1$

$\lambda_2 = -1$: $a = 0, \; b \; \forall \Rightarrow \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad b = 1$

$\left.\begin{matrix} a = 1 \\ b = 1 \end{matrix}\right\}$ зумови норм.

№12

$$V(x,t) = -xF_0 \exp\left(\frac{-t^2}{\tau^2}\right), \quad t \to -\infty$$
$$0 < x < a$$
$$|C|^2 - ?, \quad t \to \infty$$

1. $\psi^{(0)}(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi n x}{a}, \quad E_n^{(0)} = \frac{\hbar^2 \pi^2 n^2}{2ma^2}$

2. $V_{ns} = \int \psi_n^{(0)*}(x)\, V\, \psi_s^{(0)}(x)\,dx$

3. $C_n^{(1)} = \frac{1}{i\hbar} \int_{-\infty}^{+\infty} V_{ns}(\tau) e^{i\omega_{ns}\tau}\, d\tau$

$$V_{ns} = \int_0^a \frac{2}{a} \sin\frac{\pi n x}{a} \sin\frac{\pi n s}{a}\left(-xF_0 \exp\left(-\frac{t^2}{\tau^2}\right)\right)dx =$$
$$= \underbrace{-\frac{2F_0}{a}\exp\left(-\frac{t^2}{\tau^2}\right)}_{=A} \int_0^a x \sin\frac{\pi n x}{a} \sin\frac{\pi s x}{a}\,dx = A\int_0^a x \sin\frac{\pi n x}{a}\sin\frac{\pi s x}{a}\,dx$$
$$= \underbrace{\frac{A}{2}\int_0^a x\cos\left[\frac{\pi x}{a}(n-s)\right]dx}_{J_1 =} + \underbrace{\frac{A}{2}\int_0^a x\cos\left[\frac{\pi x}{a}(n+s)\right]dx}_{J_2 =} = (J_1 + J_2)\frac{A}{2}$$
$$J_1 = \left[\frac{a}{\pi(n-s)}\right]^2\left[(-1)^{n-s} - 1\right]; \quad J_2 = \left[\frac{a}{\pi(n+s)}\right]^2\left[(-1)^{n+s} - 1\right]$$
$$V_{ns} = -2\frac{F_0}{2}\exp\left(-\frac{t^2}{\tau^2}\right)\frac{a^2}{\pi^2}\left\{\frac{1}{(n-s)^2} - \frac{1}{(n+s)^2}\right\} = \frac{2aF_0}{\pi^2}\left\{\frac{1}{(n+s)^2} - \frac{1}{(n-s)^2}\right\} e^{\left(-\frac{t^2}{\tau^2}\right)} =$$
$$= B\exp\left(-\frac{t^2}{\tau^2}\right), \quad n - s = 2k+1$$
$$C_n^{(1)}(\infty) = \int_{-\infty}^{+\infty} B\exp\left(-\frac{t^2}{\tau^2}\right) e^{i\omega_{ns}t}\,dt = B\int_{-\infty}^{+\infty} e^{-(t^2 - i\omega\tau^2 t)\frac{1}{\tau^2}}\,dt =$$
$$= B\int_{-\infty}^{+\infty} e^{-\frac{1}{\tau^2}\left[\left(t - \left(\frac{i\omega\tau^2}{2}\right)\right)^2 + \frac{\omega^2\tau^4}{4}\right]}\,dt =$$
$$= B e^{-\frac{\omega^2\tau^2}{4}} \int_{-\infty}^{+\infty} e^{-\frac{1}{\tau^2}\left(t - \frac{i\omega\tau^2}{2}\right)^2}\,dt = \left[\int_{-\infty}^{+\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}\right] =$$
$$= Be^{-\frac{\omega^2\tau^2}{4}}\sqrt{\pi}\,\tau = \frac{2aF_0\tau}{\pi^{3/2}} e^{-\left(\frac{\omega\tau}{2}\right)^2}\left\{\frac{1}{(n+s)^2} - \frac{1}{(n-s)^2}\right\}, \quad n - s = 2k+1$$

Шукана ймовірність $\tilde{y}$:

$$\tilde{y} = |C_n^{(1)}(\infty)|^2 = \frac{4a^2F_0^2\tau^2}{\pi^3} e^{-\cancel{2}\frac{\omega^2\tau^2}{2}}\left\{\frac{4ns}{(n+s)^2(n-s)^2}\right\}^2$$

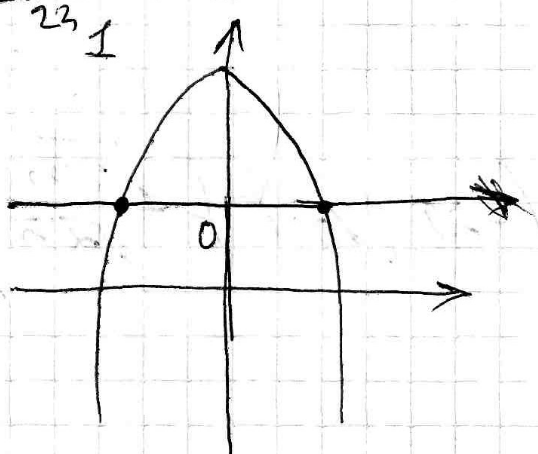
Знайти коеф. відб. R та проходження D для потенціального бар'єру №13

$$V(x) = V_0 - \alpha x^2$$

у квазікл. наближенні. Е-я час. $E < V_0$

У ВКБ

$$D = \underbrace{D_0}_{\approx 1} \exp\left[-2\frac{\sqrt{2m}}{\hbar}\int_a^b \sqrt{V(x)-E}\,dx\right]$$



$$E = V(x) \Rightarrow E = V_0 - \alpha x^2$$

$$x^2 = \frac{V_0 - E}{\alpha}$$

$$\sqrt{\frac{V_0-E}{\alpha}} = a \qquad x_{1,2} = \pm\sqrt{\frac{V_0-E}{\alpha}}$$

$$\int_{-\sqrt{\frac{V_0-E}{\alpha}} = -a}^{\sqrt{\frac{V_0-E}{\alpha}}} \sqrt{V_0 - \alpha x^2 - E}\,dx = 2\int_0^a \sqrt{V_0 - E - \alpha x^2}\,dx =$$

$$= 2\int_0^a \sqrt{V_0-E}\sqrt{1-\frac{\alpha x^2}{V_0-E}}\,dx = \left[\begin{aligned} &\frac{\alpha x^2}{V_0-E} = t \\ &x = t^{1/2}\sqrt{\frac{V_0-E}{\alpha}} \\ &dx = \frac{1}{2}t^{-1/2}\sqrt{\frac{V_0-E}{\alpha}} \end{aligned}\right] =$$

$$= 2\sqrt{V_0-E}\,\frac{1}{2}\sqrt{\frac{V_0-E}{\alpha}}\int_0^1 (1-t^2)\,t^{\frac{1}{2}}\,dt$$

$$= \frac{V_0-E}{\sqrt{\alpha}} B\left(\frac{3}{2};\frac{1}{2}\right) = \frac{V_0-E}{\sqrt{\alpha}}\,\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(2)} = \frac{\pi}{2}\,\frac{V_0-E}{\sqrt{\alpha}}$$

$$D = D_0 \exp\left(-2\frac{\sqrt{2m}}{\hbar} \cdot \frac{V_0 - E}{\sqrt{\alpha}} \frac{\pi}{2}\right) =$$

$$= D_0 \exp\left(-\frac{\sqrt{2m}\,\pi}{\hbar} \frac{V_0 - E}{\sqrt{\alpha}}\right)$$

$$D = D_0 \exp\left(-\pi (V_0 - E) \frac{\sqrt{2m}}{\sqrt{\alpha}\,\hbar}\right)$$

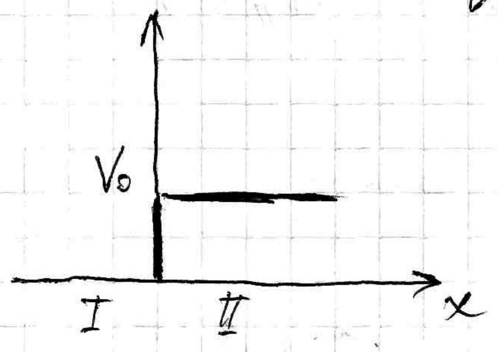
$$R = 1 - D$$

$$R = 1 - D_0 \exp\left(-\pi (V_0 - E) \frac{\sqrt{2m}}{\sqrt{\alpha}\,\hbar}\right)$$

Знайти коеф. відбиття та проходження для потенц. бар'єру №14

№15

$$V(x) = \begin{cases} 0, & x<0 \\ V_0, & x>0 \end{cases}$$



$$I \quad -\frac{\hbar^2}{2\mu}\frac{d^2\psi}{dx^2} = E\psi$$

$$\psi'' = -k^2\psi, \quad k^2 = \frac{2\mu E}{\hbar^2}$$

$$\psi(x) = A_1 e^{ikx} + A_2 e^{-ikx}$$

$$j_x = \frac{i\hbar}{2\mu}\left\{\psi\frac{d\psi^*}{dx} - \psi^*\frac{d\psi}{dx}\right\} =$$

$$= \frac{i\hbar}{2\mu}\left\{A_1 e^{ikx}(-ik)A_1^* e^{-ikx} - ik|A_1|^2\right\} =$$

$$= \frac{\hbar k}{\mu}|A_1|^2$$

$$\psi_I(x) = \underbrace{A_1 e^{ikx}}_{\text{рух вправо}} + \underbrace{A_2 e^{-ikx}}_{\text{рух вліво}}$$

$$II \quad -\frac{\hbar^2}{2\mu}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\psi'' = \underbrace{-\frac{2\mu(E-V_0)}{\hbar^2}}_{=\varkappa^2>0}\psi$$

$$\psi_{II}(x) = c_1 e^{i\varkappa x} + \underset{0}{c_2} e^{-i\varkappa x}$$

Г.У.

$$\begin{cases} \psi_I(0) = \psi_{II}(0) \\ \psi_I'(0) = \psi_{II}'(0) \end{cases} \Leftrightarrow \frac{\psi_I'(0)}{\psi_I(0)} = \frac{\psi_{II}'(0)}{\psi_{II}(0)}$$

$$\begin{cases} A_1 + A_2 = C_1 \\ k(A_1 - A_2) = i\varkappa C_1 \end{cases}$$

$$D = \frac{|j_{пр}|}{|j_{пад}|} = \frac{\frac{\hbar\varkappa}{\mu}|C_1|^2}{\frac{\hbar k}{\mu}|A_1|^2} = \frac{\varkappa}{k}\frac{|C_1|^2}{|A_1|^2}$$

$$1 + \frac{A_2}{A_1} = \frac{C_1}{A_1}$$

$$\frac{A_2}{A_1} = \frac{C_1}{A_1} - 1$$

$$\frac{C_1}{A_1} = \frac{2k}{\varkappa + k}$$

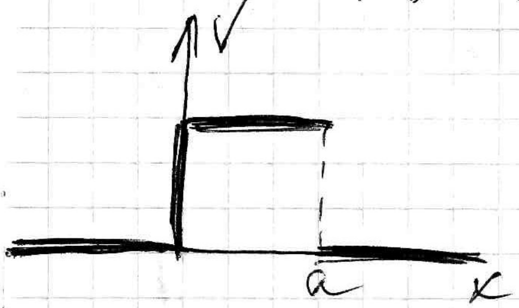
$$D = \left(\frac{2k}{\varkappa+k}\right)^2 \frac{\varkappa}{k} = \frac{4k\varkappa}{(\varkappa+k)^2}$$

коеф. відбиття

$$R = 1 - D = 1 - \frac{4k\varkappa}{(\varkappa+k)^2} = \frac{(\varkappa-k)^2}{(\varkappa+k)^2}$$

№15

У ВКБ наближенні оцінити D через одновимірний потенціал



$$V(x) = \begin{cases} 0; & x<0, x>a \\ V_0; & 0 \le x \le a \end{cases}$$

$$D = D_0 \exp\left[-2\frac{\sqrt{2m}}{\hbar}\int_a^b \sqrt{V(x)-E}\,dx\right]$$

$$\int_0^a \sqrt{V_0-E}\,dx = a\sqrt{V_0-E}$$

$$D = D_0 \exp\left[-2\frac{\sqrt{2m}}{\hbar}\left(a\sqrt{V_0-E}\right)\right]$$

$$R = 1-D = 1 - D_0 \exp\left[-2\frac{\sqrt{2m}}{\hbar}\left(a\sqrt{V_0-E}\right)\right]$$