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Домашнє №7

№ 375

$F[\operatorname{sign} x] = 2i \cdot P\frac{1}{x}$

$\operatorname{sign} x = \theta(x) - \theta(-x)$ => Шукаємо

Фур'є образ

$F[\operatorname{sign} x] = F[\theta(x)] - F[\theta(-x)]$.

$F[\theta(-x)\, e^{\varepsilon x}]$ $\varepsilon > 0$ – константа

$(F[\theta(-x)e^{\varepsilon x}]; \varphi) = (\theta(-x)e^{\varepsilon x}, F[\varphi])$

$= \int_{-\infty}^{0} dx\, e^{\varepsilon x} \int_{-\infty}^{+\infty} dk\, \varphi(k)\, e^{ikx} = \int_{-\infty}^{\infty} dk\, \varphi(k)\, \mathcal{I}(k) =$

$= (\mathcal{I}, \varphi)$;

$\mathcal{I}(k) = \int_{-\infty}^{0} dx\, e^{ikx + \varepsilon x} \cdot \int_{-\infty}^{0} dx\, e^{i(k - i\varepsilon)x} =$

$= -\frac{i}{k - i\varepsilon}$;

Звідси маємо:

$F[\theta(-x)\, e^{\varepsilon x}] = -\frac{i}{k - i\varepsilon}$,

Тепер перейдемо до границі $\varepsilon \to +0$;

$F[\theta(-x)] = \lim_{\varepsilon \to +0} \left(-\frac{i}{k - i\varepsilon}\right) = -\frac{i}{k - i0} = \pi\delta(k) -$
$- i \mathcal{P}\frac{1}{k};$

Аналогічно до 8.2 (приклад) з лекції

$F[\theta(x)] = \lim_{\varepsilon \to +0} \frac{i}{k + i\varepsilon} = + \frac{i}{k + i0} = i\mathcal{P}\frac{1}{k} +$
$+ \pi\delta(k);$

$F[\operatorname{sign} x] = F[\theta(x)] - F[\theta(-x)] = \pi\delta(k) +$
$+ i\mathcal{P}\frac{1}{k} - \pi\delta(k) + i\mathcal{P}\frac{1}{k} = 2i\mathcal{P}\frac{1}{k}$, що і треба було довести

№ 381

$F[x_+] = -\frac{1}{(k + i0)^2}$

$x_+ = x\theta(x) = \theta(x) * \theta(x)$

За властивістю перетворення Фур'є:

$F[f * g] = F[f]F[g] \Rightarrow$

$\Rightarrow F[x_+] = F[\theta(x)]F[\theta(x)] =$

$= \frac{i}{k + i0} \cdot \frac{i}{k + i0} = \frac{i^2}{(k + i0)^2} = -\frac{1}{(k + i0)^2}$

№397

$x^n u(x) = 0$

$F[x^n u(x)] = 0;$

$$\partial^{\alpha} F[\varphi] = F[(ix)^{\alpha}\varphi]$$

$$F[(ix)^{n} u(x)] = \partial^{n} F[u] = 0 \Rightarrow$$

$$\Rightarrow F[u] = C_1 + C_2 k + C_3 \frac{k^2}{2} + C_4 \frac{k^3}{6} \ldots \ominus$$

$$= F[u] = a_0 + a_1 k + \ldots + a_{n-1} k^{n-1},$$

$$u = F[1] a_0 + \ldots + a_{n-1} F^{-1}[\cancel{k^{n-1}}]$$

$$\ominus \sum_{m=1}^{n} C_m \frac{k^{m-1}}{(m-1)!} = \sum_{m=1}^{n} C_m k^{m-1}$$

$$u(x) = F^{-1}[C_1 + C_2 k + C_3 \frac{k^2}{2} + C_4 \frac{k^3}{6} + \ldots] =$$

$$= C_1 F^{-1}[1] + C_2 F^{-1}[k] + C_3 F^{-1}[k^2] +$$

$$+ C_4 F^{-1}[k^3] + \ldots = C_1 \delta(x) - iC_2 \delta'(x) -$$

$$C_3 \delta''(x) + i C_4 \delta'''(x) + \ldots =$$

$$= C_0 \delta(x) - iC_1 \delta'(x) - C_2 \delta''(x) + i C_3 \delta'''(x) +$$

$$+ \ldots = \sum_{k=0}^{n} C_k (-i)^k \delta^{(k)}(x).$$

N399

$$x^n \frac{d^m y(x)}{dx^{(m)}} = 0 \qquad n > m$$

$$\left(F\left[x^n y^{(m)}(x)\right], \varphi(k)\right) = \left(x^n y^{(m)}(x), F[\varphi(k)]\right) =$$

$$= \left(y^m(x), i^n(-ix)^n F[\varphi(k)]\right) = \left(y^m(x), i^n \cdot F\left[\frac{\partial^n}{\partial k^n}\varphi(k)\right]\right) = \left(\frac{\partial^n}{\partial k^n}\varphi(k), i^n F\left[y^m(x)\right]\right) =$$

$$= \left((-i)^n \frac{\partial^n}{\partial k^n} F\left[y^{(m)}(x)\right], \varphi(k)\right) \Rightarrow$$

$$\Rightarrow \frac{\partial^n}{\partial k^n} F\left[y^m(x)\right] = 0$$

після n-кратного інтегрування:

$$F\left[y^{(m)}(x)\right] = b_0 + b_1 k + \ldots + b_{n-1} k^{n-1};$$

$$F^{-1}\left[k^n\right] = i^n \delta^{(n)}(x) \Rightarrow$$

$$\Rightarrow y^{(m)}(x) = b_0 F^{-1}[1] + b_1 F^{-1}[k] + \ldots + b_{n-1} F^{-1}\left[k^{n-1}\right] = y^m(x) = b_0 \delta(x) + c_1 \delta'(x) + \ldots + c_{n-1}\delta^{(n-1)}(x)$$

$$y^{(m-1)}(x) = b_0 \theta(x) + C_1 \delta(x) + \ldots + C_{n-1}\delta^{(n-2)}(x) + a_{m-1}$$

a_{m-1} – const

$$y^{(m-2)}(x) = b_0 x\theta(x) + c_1\theta(x) + c_2\delta(x) + \ldots + c_{n-1}\delta^{(n-3)}(x) + a_{m-1}x + a_{m-2};$$

$$y^{(m-3)}(x) = \frac{b_0}{2}x^2\theta(x) + c_1 x\theta(x) + c_2\theta(x) + c_3\delta(x) + \ldots + c_{n-1}\delta^{(n-4)}(x) + a_{m-1}\frac{x^2}{2} + a_{m-2}x + a_{m-3};$$

$$y(x) = \sum_{k=0}^{m-1} a_k x^k + \sum_{k=0}^{m-1} b_k\theta(x)x^{m-k-1} + \sum_{k=m}^{n-1} c_k\delta^{(k-m)}(x).$$