

Домашнее задание №6

N325

$$\delta(\cos x)$$

$$\delta(y(x)) = \sum_{k=-\infty}^{\infty} [\delta(x - x_k) |y'(x_k)|]$$

x_k - все корни уравнения $y(x) = 0$

$$\cos x = 0$$

$$x_k = \frac{\pi}{2} + k\pi, \text{ где } k = 0, \pm 1, \dots$$

$$|y'(x_k)| = 1;$$

$$\delta(\cos x) = \sum_{k=-\infty}^{\infty} \delta(x - \frac{\pi}{2} + k\pi)$$

N328

$$x^m \rho \frac{1}{x} = x^{m-1}$$

$$\begin{aligned} (x^m \rho \frac{1}{x}, \varphi) &= (\rho \frac{1}{x}, x^m \rho) = \int \frac{x^m}{x} \varphi(x) dx = \\ &= (x^{m-1}, \varphi(x)) \end{aligned}$$

N329

$$x^n \delta(x) = 0, \text{ где } n \in \mathbb{N}$$

Несомненно $\varphi \in D$, тогда

$$\begin{aligned} ((x^n \delta(x), \varphi(x)) &= (\delta(x), x^n \varphi(x)) = x^n \varphi(0) = \\ &= 1 \varphi(0) = 1 \delta(x), \varphi(x) \Rightarrow 0 \end{aligned}$$

N332

$$\int (x^3 - 7x + 6) = \frac{1}{4} \delta(x-1) + \frac{1}{5} \delta(x-2) + \frac{1}{20} \delta(x+3)$$

$$x^3 - 7x + 6 = 0$$

$$x_k \Rightarrow \begin{aligned} x_1 &= -3 \\ x_2 &= 1 \\ x_3 &= 2 \end{aligned}$$

Таким образом $\int (x^3 - 7x + 6) = \frac{1}{7} \delta(x-1) + \frac{1}{5} \delta(x-2) + \frac{1}{20} \delta(x+3)$.

N336

$$y(x) = \begin{cases} |\sin x|, & |x| \leq \pi \\ 0, & |x| \geq \pi \end{cases}$$

$$y(x) = \Theta(\pi - |x|) \sin x \operatorname{sign} x = \Theta(\pi - |x|) \cdot$$

$$\cdot \sin x [\Theta(x) - \Theta(-x)] :$$

$$y'(x) = 2\delta(x) \Theta(\pi - |x|) \sin x + \Theta(\pi - |x|) \operatorname{sign} x \cdot$$

$$\cdot \cos x + \sin x \operatorname{sign} x \Theta(\pi - |x|)'$$

$$(\Theta(\pi - |x|)', \varphi(x)) = -(\Theta(\pi - |x|), \varphi'(x)) =$$

$$= -\int_{-\pi}^{\pi} \varphi'(x) dx = \varphi(-\pi) - \varphi(\pi) = \int_{-\pi}^{\pi} \delta(x+\pi) \cdot$$

$$\cdot \varphi(x) dx - \int_{-\pi}^{\pi} \delta(x-\pi) \varphi(x) dx = (\delta(x+\pi) - \delta(x-\pi), \varphi(x)).$$

$$y'(x) = \Theta(\pi - |x|) \operatorname{sign} x \cos x + \sin x \operatorname{sign} x.$$

$$\begin{aligned} \cdot [\delta(x+\pi) - \delta(x-\pi)] &= \Theta(\pi - |x|) \operatorname{sign} x \cos x \\ &= \Theta(\pi - |x|) [\Theta(x) - \Theta(-x)] \cos x; \end{aligned}$$

$$y''(x) = -\sin x \Theta(\pi - |x|) [\Theta(x) - \Theta(-x)] + 2\delta(x) \cos x \Theta(\pi - |x|) + \cos x \operatorname{sign} x.$$

$$\begin{aligned} \cdot [\delta(x+\pi) - \delta(x-\pi)] &= -\sin x \Theta(\pi - |x|) \operatorname{sign} x \\ &+ 2\delta(x) - \delta(x+\pi) + \delta(x-\pi). \end{aligned}$$

$$y^n(x) = \Theta(\pi - |x|) \operatorname{sign} x \sin^{(n)} x - \sum_{k=1}^{n/2} (-1)^k.$$

$$\cdot \left\{ 2\delta^{(n-2k)}(x) - \delta^{(n-2k)}(x+\pi) + \delta^{(n-2k)}(x-\pi) \right\}$$

$$n = 2, 3, \dots$$

N 343

$$u(x) = x^2 \delta'(x)$$

$$\begin{aligned} (u(x), \varphi) &= (x^2 \delta', \varphi) = (\delta', x^2 \varphi) = \\ &= -(\delta, (x^2 \varphi)') = -(\delta, \varphi + 2x\varphi') = \\ &= -(\delta, \varphi) - (\delta, 2x\varphi') = -\varphi(0) - 2x\varphi' \Big|_{x=0} = \\ &= -\varphi(0) - (-\delta, \varphi). \end{aligned}$$

N 344

$$y(x) = x^2 \delta''(x);$$

$$\begin{aligned}
 (x^2 f''(x), \varphi(x)) &= (f''(x), x^2 \varphi(x)) = \\
 &= (f(x), [x^2 \varphi(x)]'') = (f(x), 2\varphi(x) + \\
 &+ 4x\varphi'(x) + x^2 \varphi''(x)) = (2f(x), \varphi(x)); \\
 y(x) &= 2f(x).
 \end{aligned}$$