Домашняя работа №6

№325

$\delta(\cos x)$

$$\delta(y(x)) = \sum_{k=-\infty}^{\infty} \left[\delta(x - x_k) \cdot |y'(x_k)|\right]$$

x_k – розв. рівняння $y(x) = 0$

$\cos x = 0$

$x_k = \frac{\pi}{2} + k\pi$, де $k = 0, \pm 1 \dots$

$|y'(x_k)| = 1$;

$$\delta(\cos x) = \sum_{k=-\infty}^{\infty} \delta\left(x - \frac{\pi}{2} + k\pi\right)$$

№328

$x^m \rho \frac{1}{x} = x^{m-1}$

$$(x^m \rho \tfrac{1}{x}, \varphi) = (\rho \tfrac{1}{x}, x^m \rho) = \int \frac{x^m}{x} \varphi(x)\,dx =$$

$$= (x^{m-1}, \varphi(x))$$

№329

$x^n \delta(x) = 0$, де $n \in \mathbb{N}$

Нехай $\varphi \in D$, тоді

$$((x^n \delta(x), \varphi(x)) = (\delta(x), x^n \varphi(x)) = x^n \varphi(0) =$$

$$= 1\varphi(0) = 1\delta(x), \varphi(x) \Rightarrow 0;$$

№332

$$\delta(x^3-7x+6)=\frac{1}{4}\delta(x-1)+\frac{1}{7}\delta(x-2)+\frac{1}{20}\delta(x+3)$$

$x^3-7x+6=0$

$x_k \Rightarrow$ $x_1=-3$, $x_2=1$, $x_3=2$

Таким числом $\delta(x^3-7x+6)=\frac{1}{7}\delta(x-1)+\frac{1}{5}\delta(x-2)+$ $+\frac{1}{20}\delta(x+3)$.

№336

$$y(x)=\begin{cases}|\sin x|, & |x|\le\pi\\ 0, & |x|\ge\pi\end{cases}$$

$y(x)=\theta(\pi-|x|)\sin x\,\mathrm{sign}\,x=\theta(\pi-|x|)\cdot$ $\cdot\sin x[\theta(x)-\theta(-x)]$:

$y'(x)=2\delta(x)\theta(\pi-|x|)\sin x+\theta(\pi-|x|)\mathrm{sign}\,x\cdot$ $\cdot\cos x+\sin x\,\mathrm{sign}\,x\,\theta(\pi-|x|)'$

$(\theta(\pi-|x|)',\varphi(x))=-(\theta(\pi-|x|),\varphi'(x))=$

$=-\int_{-\pi}^{\pi}\varphi'(x)\,dx=\varphi(-\pi)-\varphi(\pi)=\int_{-\pi}^{\pi}\delta(x+\pi)\cdot$ $\cdot\varphi(x)\,dx-\int_{-\pi}^{\pi}\delta(x-\pi)\varphi(x)\,dx=(\delta(x+\pi)-$ $-\delta(x-\pi),\varphi(x))\cdot$

$$y'(x) = \theta(\pi - |x|)\,\mathrm{sign}\,x\cos x + \sin x\,\mathrm{sign}\,x \cdot [\delta(x+\pi) - \delta(x-\pi)] = \theta(\pi-|x|)\,\mathrm{sign}\,x\cos x = \theta(\pi-|x|)[\theta(x) - \theta(-x)]\cos x;$$

$$y''(x) = -\sin x\,\theta(\pi-|x|)[\theta(x) - \theta(-x)] + 2\delta(x)\cos x\,\theta(\pi-|x|) + \cos x\,\mathrm{sign}\,x \cdot [\delta(x+\pi) - \delta(x-\pi)] = -\sin x\,\theta(\pi-|x|)\,\mathrm{sign}\,x + 2\delta(x) - \delta(x+\pi) + \delta(x-\pi).$$

$$y^{(n)}(x) = \theta(\pi-|x|)\,\mathrm{sign}\,x \sin^{(n)} x - \sum_{k=1}^{n/2} (-1)^k \cdot \{2\delta^{(n-2k)}(x) - \delta^{(n-2k)}(x+\pi) + \delta^{(n-2k)}(x-\pi)\}$$

$n = 2, 3 \ldots$

№ 343

$$u(x) = x^2\delta'(x)$$

$$(u(x), \varphi) = (x^2\delta', \varphi) = (\delta', x^2\varphi) = -(\delta, (x^2\varphi)') = -(\delta, \varphi + 2x\varphi') = -(\delta, \varphi) - (\delta, 2x\varphi') = -\varphi(0) - 2x\varphi'\Big|_{x=0} = -\varphi(0) = (-\delta, \varphi).$$

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$$y(x) = x^2\delta^{(4)}(x);$$

$$(x^2\delta''(x), \varphi(x)) = (\delta''(x), x^2\varphi(x)) =$$
$$= (\delta(x), [x^2\varphi(x)]'') = (\delta(x), 2\varphi(x) +$$
$$+ 4x\varphi'(x) + x^2\varphi''(x)) = (2\delta(x), \varphi(x)),$$
$$y(x) = 2\delta(x).$$