Домашня робота №4

№ 188

$$\begin{cases} \Delta U(x,y)=0 \\ U(0,y)=U_1 \\ U(a,y)=U_2 \\ U(x,0)=0 \\ U(x,b)=0 \end{cases}$$

I
$$\begin{cases} \Delta U(x,y)=0 \\ U(x,0)=0 \\ U(x,b)=0 \\ U(0,y)=U_1 \\ U(a,y)=0 \end{cases}$$

$U(x,y)=\sum X_n(x)Y_n(y);$

$X_n''Y_n+X_nY_n''=0;$

$$\frac{X_n''}{X_n}=-\frac{Y_n''}{Y_n}=\lambda_n^2;$$

II
$$\begin{cases} \Delta U(x,y)=0 \\ U(x,0)=0 \\ U(x,b)=0 \\ U(0,y)=0 \\ U(a,y)=U_2 \end{cases} \Rightarrow \begin{cases} X_n''-\lambda_n^2X_n \\ X(0)=U_1 \\ X(a)=0 \end{cases}$$

$k^2-\lambda^2=0;$

$k=\pm\lambda$

$X_n(x)=C_{1n}e^{\lambda_n x}+C_{2n}e^{-\lambda_n x};$

$$u(x,y)=0=\sum_{n+1}^{\infty}\sin\lambda_n y\left(C_{1n}e^{\lambda_n x}+C_{2n}e^{-\lambda_n x}\right).$$

$$C_{1n}e^{\lambda_n x}+C_{2n}e^{-\lambda_n x}=0 \Rightarrow U(0,y)=\sum_{n+1}^{\infty}\sin\lambda_n y\cdot$$

$$\cdot\left(C_{1n}-C_{1n}e^{-2\lambda_n x}\right)=U_1;$$

$$\sum C_{1n}\sin\lambda_n y\left(1-e^{2\lambda_n}\right)=U_1;$$

$$C_{1n} = \frac{V_1}{1-e^{2\lambda_n a}}$$

$$u^{I}(x,y) = \sum_{n=1}^{\infty} \sin\frac{\pi n y}{6} \frac{\mathcal{V}_1}{(1-e^{2\lambda_n a})} \cdot$$

$$\cdot \left(e^{\pi x} - e^{\wedge}\frac{2\pi a n}{\ell} x\right);$$

$$u(x,y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin\frac{\pi n y}{6}}{\operatorname{sh}\frac{\pi a y}{6}} \left[1-(-1)^n\right] \cdot$$

$$\cdot \left[V_2 \operatorname{sh}\frac{\pi n x}{b} + V_1 \operatorname{sh}\frac{\pi n}{b}(a^6 - x)\right].$$

N 199

$$u_\rho(R,\varphi) = A_0 + B\cos^3\varphi$$

$$\Delta u = u_{\rho\rho} + \frac{1}{\rho}u_\rho + \frac{1}{\rho^2}u_{tt} = 0;$$

$$u^{int}(\rho) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(\frac{\rho}{R}\right)^n \left(A_n\cos(n\varphi) - B_n\sin(n\varphi)\right)$$

$$u^{ext}(\rho) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(\frac{R}{\rho}\right)^n \left(A_n\cos(n\varphi) + B_n\sin(n\varphi)\right)$$

$$u_0 + u_1\cos 2\varphi = u_\rho;$$

$$u_\rho^{int} = \sum_{n=1}^{\infty}\frac{n\rho^{n-1}}{R^n}\left(A_n\cos(n\varphi) + B_n\sin(n\varphi)\right)$$

$$A + B\cos^{P}\varphi = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(A_n\cos(n\varphi) + B_n\sin(n\varphi)\right)$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} f(\varphi)\cos n\varphi\, d\varphi;$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} f(\varphi)\sin n\varphi\, d\varphi;$$

$$\cos^2\varphi = \frac{1}{4}(3\cos\varphi + 3\cos^3\varphi) = \frac{3}{4}\cos\varphi + \frac{1}{4}\cos 3\varphi;$$

$n = 1 \Rightarrow A_1 = \frac{3}{4}; \; B_1 = 0;$

$n = 3 \Rightarrow A_3 = \frac{1}{4}; \; B_3 = 0;$

$A_n = 0; \; B_n = 0; \; A_0 = 2A;$

$$u^{int}(\rho,\varphi) = A + \left(\frac{\rho}{R}\right)\frac{3}{4}\cos\varphi + \left(\frac{\rho}{R}\right)^3\frac{1}{4}\cos^3\varphi$$

$$u^{ext}(\rho,\varphi) = A + \left(\frac{R}{\rho}\right)\frac{3}{4}\cos\varphi + \left(\frac{\rho}{R}\right)^3\frac{1}{4}\cos^3\varphi$$

$$u(\rho,\varphi) = \frac{A_0}{2} - \frac{3}{4}\frac{R^2}{\rho}\cos\varphi - \frac{8R^4}{12\rho^3}\cos 3\varphi.$$

N 224

$$\begin{cases} \Delta u = 0 \\ u(x,y) = x^2 - y^2 \end{cases}$$

$x^2 + y^2 = R^2$

$\rho = R;$

$\rho^2\cos^2\varphi - \rho^2\sin^2\varphi = 0 = \rho^2\cos^2\varphi = R^2\cos 2\varphi$

$u(R,\varphi) = R^2\cos 2\varphi$

$$R^2\cos 2\varphi = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n\cos n\varphi + B_n \sin n\varphi;$$

$A_n = 0$

$n = 2 \Rightarrow A_n = 1; \; B_n = 0$

$u^{int}(\rho,\varphi) = \rho^2\cos 2\varphi; \; u^{ext}(\rho,\varphi) = \frac{R^2}{\rho^2}\cos 2\varphi.$

N209

$$\left\|\begin{array}{l}\Delta u = 0 \\ u(R,\varphi) = h u_\rho(R,\varphi) = A \sin n\varphi\end{array}\right.$$

$$u^{int} = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left(\frac{\rho}{R}\right)^n (A_n \cos n\varphi + B_n \sin n\varphi)$$

$$u^{ext} = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left(\frac{R}{\rho}\right)^n (A_n \cos n\varphi + B_n \sin n\varphi)$$

$$u_\rho^{int} = \sum_{n=1}^{\infty} n \frac{\rho^{n-1}}{R^n} (A_n \cos n\varphi + B_n \sin n\varphi)$$

$$u_\rho^{ext} = \sum_{n=1}^{\infty} (-n) \frac{R^n}{\rho^{n-1}} (A_n \cos \varphi n + B_n \sin n\varphi).$$

$$u^{int}(R,\varphi) + \cancel{h} u_\rho^{int}(R,\varphi) = \frac{A_0}{2} + \sum_{n=1} (A_n \cos n\varphi + B_n \sin n\varphi)\left(1 + \frac{hn}{R}\right) = u_1 \sin n\varphi;$$

$$n = 4; \quad B_4\left(1 + \frac{4}{R}\right) = u_1 \Rightarrow B_n = \frac{u_1 R}{4h + R};$$

$$\{A_n = 0, \ n \in \mathbb{Z}\}$$

$$u^{int} = \left(\frac{\rho}{R}\right)^4 \frac{u_1 R}{4 + R} \sin 4\varphi;$$

$$u^{ext}(R,\varphi) + h u_\rho^{ext}(R,\varphi) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\varphi + B_n \sin n\varphi)(1 - hnR)$$

$$n = 4, \quad B_n = \frac{u_1}{1 - 4hR} \quad \{A_n = 0, \ n \in \mathbb{Z}\};$$

$$u^{ext} = \frac{u_1 \sin 4\varphi}{1 - 4hR} \left(\frac{R}{\rho}\right)^4.$$

№235

$$\begin{cases} u_\rho(R_1, \varphi) = \sin\varphi, \\ u_\rho(R_2, \varphi) = \cos\varphi \end{cases}$$

$$u(\rho, \varphi) = A_0 \ln\rho + B_0 + \sum_{m=1}^{\infty} \left(A^{I}_m \rho^{-m} + B^{I}_m \rho^{m}\right) \cdot$$
$$\cdot \cos m\varphi + \sum_{m=1}^{\infty} \left(A^{II}_m \rho^{-m} + B^{II}_m \rho^{m}\right) \cdot \sin m\varphi.$$

$$u_\rho = \frac{A_0}{\rho} + \sum_{m=1}^{\infty} \left(-m A^{I}_m \rho^{-m-1} + m B^{I}_m \rho^{m-1}\right) \cos m\varphi +$$
$$+ \sum_{m=1}^{\infty} \left(-m A^{II}_m \rho^{-m+1} + m B^{II}_m \rho^{m-1}\right) \sin m\varphi ;$$

$$\Rightarrow \frac{A_0}{R} + \sum_{m=1}^{\infty} \left(-m A^{I}_m R_1^{-m-1} + m B^{I}_m R_1^{m-1}\right) \cos m\varphi +$$
$$+ \sum_{m=1}^{\infty} \left(-m A^{II}_m R_1^{-m+1} + m B^{II}_m R_1^{m-1}\right) \sin m\varphi = \sin\varphi$$

$A_0 = 0$

$B_0 = 0$

$\left(B^{II}_1 - A^{II}_1 R_1^{-2}\right) \sin\varphi = \sin\varphi ;$

$B^{II}_1 - A^{II}_1 R_1^{-2} = 1 ;$

$B^{I}_1 - A^{I}_1 R_1^{-2} = 0 ;$

$B^{I}_1 - A^{I}_1 R_2^{-2} = 1 ;$

$B^{II}_1 - A^{II}_1 R_2^{-2} = 0 ;$

$$-A_1^{II} R_1^{-2} + A_1^{II} R_2^{-2} = 1;$$

$$A_1^{II} = \frac{1}{R_2^{-2} - R_1^{-2}}; \quad B_1^{II} = \frac{1}{1-(R_2/R_1)^2};$$

$$-A_1^{I} R_2^{-2} + A_1^{I} R_1^{-2} = 1$$

$$A_1^{I} = \frac{1}{R_1^{-2} - R_2^{-2}}; \quad B_1^{I} = \frac{1}{1-(R_1/R_2)^2}.$$