Домашня робота №3

№ 98

$$\begin{cases} u_t = u_{xx} + u + 2\cos 3x \\ u_x(0,t) = u_x(\pi,t) = 0 \\ u(x,0) = \cos x \end{cases}$$

$u(x,t) = X(x)\,T(t)$

$X'' + \lambda^2 X = 0;$

$X'(0) = X'(\pi) = 0;$

$X_n(x) = \cos \lambda_n x$; $\lambda_n = n$, де $n = 0, 1, 2 \ldots$

$T_n' + (n^2-1)T_n = f_n$, де $f_n = 2\delta_{n,3}$ – коефіцієнт розкладу ф-ї $f(x,t) = 2\cos 3x$;

$n > 3$

$T_0' - T_0 = 0;$

$T_1' = 0;$

$T_2' + 3T_2 = 0;$

$T_n' + (n^2-1)T_n = 0$

$T_0(t) = C_0 \exp t;$

$T_1(t) = C_1$; $T_2(t) = C_2 \exp(-3t)$;

$T_n(t) = C_n \exp[(1-n^2)t]$.

$T_3' + 8T_3 = 2; \Rightarrow T_3(t) = C_3 \exp(-8t) + \frac{1}{4};$

$u(x,t) = C_0 \exp t + C_1 \cos x + C_2 \exp(-3t) \cdot$
$\cdot \cos 2x + \left[C_3 \exp(-8t) + \frac{1}{4}\right] \cos 3x +$
$+ \sum_{n=4}^{\infty} C_n \exp\left[(1-n^2)t\right] \cos nx.$

$u(x,0) = C_0 + C_2 \cos 2x + \left[C_3 + \frac{1}{4}\right] \cos 3x +$
$+ \sum_{n=4}^{\infty} C_n \cos nx = \cos x;$

$C_0 = C_2 = 0;$

$C_1 = 1;$

$C_3 = -\frac{1}{4}$

$C_n = 0$, при $n \geq 4$

$u(x,t) = \cos x + \left[1 - \exp(-8t)\right] \cos 3x/4.$

Перевірка:

$u(x,0) = \cos x; \quad u_t = \exp(-8t) \cos 3x/2;$

$u_x = -\sin x - 3\left[1 - \exp(-8t)\right] \sin 3x/4;$

$u_{xx} = -\cos x - 9\left[1 - \exp(-8t)\right] \cos 3x/4;$

$u_{xx} + u + 2\cos 3x = -\cos x - 9\left[1 - \exp(-8t)\right] \cdot$
$\cdot \cos 3x/4 + \cos x + \left[1 - \exp(-8t)\right] \cos 3x/4 +$
$+ 2\cos 3x = u_t.$

N 113

$$\begin{cases} u_t = u_{xx} + u - x + 1 - u\cos^3\frac{5\pi x}{2} \\ u_x(0,t) = 1 \\ u(1,t) = 0 \\ u(x,0) = x - 1 \end{cases}$$

$u(x,t) = 5(x,t) + \omega(x,t)$

$$\begin{cases} \vartheta(0,t) = 1 \\ \vartheta(1,t) = 0 \end{cases}$$

$u(x,t) = \vartheta(x,t) + \omega(x,t) = x - 1 + \omega(x,t)$

$\omega_t = \omega_{xx} - 1 + x + \omega - x + 1 + (1 - x + \omega)\cos^3\frac{5\pi x}{2}$.

N 141

$$\begin{cases} u_{tt} = a^2 u_{xx} \\ u(0,t) = u_x(l,t) = 0 \\ u(x,0) = 1 \\ u_t(x,0) = \sin\frac{\pi x}{2l} \end{cases}$$

$u(x,t) = \sum T_n(t) X_n(x);$

$T''X = a^2 X'' T;$

$\frac{T''}{a^2 T} = \frac{X''}{X} = -\lambda^2;$

$X_n \cancel{+} \lambda^2 X_n = 0;$

$$X_n = A_n \sin \lambda_n x + B_n \cos \lambda_n x \quad \Rightarrow B_n = 0$$

$$X_n' = \lambda_n A_n \cos \lambda_n x - B_n \lambda_n \sin \lambda_n x \Rightarrow A_n \lambda_n \cos \lambda_n l =$$

$$= 0 \Rightarrow \cos \lambda_n l = 0 ;$$

$$\begin{cases} \lambda_n = \frac{\pi}{2l}(2n-1) \\ X_n = A_n \sin \lambda_n x \end{cases} \quad n \in \mathbb{Z}$$

$$T'' + a^2 \lambda^2 T = 0$$

$$T_n = \tilde{A}_n \cos(a, \lambda, t) + \tilde{B}_n \sin(a, \lambda_n, t)$$

$$T_n' = \tilde{B}_n \lambda_n \cos(a, \lambda, t) - \tilde{A}_n \lambda_n \sin(a, \lambda_n, t)$$

$$T_n(0) = 1 \Rightarrow \tilde{A}_n = 1$$

$$T_n'(0) = 1 \Rightarrow \tilde{B}_n = 1$$

$$u(x,t) = \sum_{n=1}^{\infty} A_n T_n X_n = \sum_n A_n \sin \lambda_n x (\sin(a,$$

$$\lambda, t) = \sum_n A_n \sin(\lambda, n, x) \sin(a, \lambda, t)$$

$$u_t(x,t) = A_n a \lambda_n \sin \lambda_n x \cos(a) = \sin \frac{\pi x}{2l} ;$$

$$\lambda_n = \frac{\pi}{2l}(2n-1) ;$$

$$A_n = \frac{2l}{\pi} ;$$

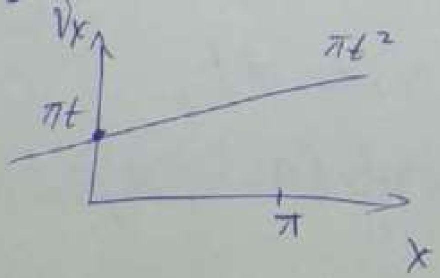
$$u(x,t) = \frac{2l}{\pi a} \sin\left(\frac{\pi x}{2l}\right) \sin\left(\frac{\pi a}{2l} t\right).$$

№175

$$\begin{cases} u_{tt} = 16u_{xx} + u + t\cos x - \pi t x + \frac{x^2}{2}\left(2 - t - t^2\right) - 16\left(t^2 - t + 1\right) \\ u_x(0,t) = \pi t \\ u_x(\pi,t) = \pi t^2 \\ u(x,0) = 16 + 2\cos x \\ u_t(x,0) = \pi x - \frac{x^2}{2} + \cos 2x \end{cases}$$

$V_x(0,t) = \pi t$

$V_x(\pi,t) = \pi t^2$



$V_x(x,t) = a + bx$

$V_x(0,t) = \pi t = a$,

$V_x(\pi,t) = \pi t^2 = a + b\pi$;

$b = t^2 - t$

$V_x = \pi t + (t^2 - t)x$.

$V(x,t) = \pi t x + (t^2 - t)\frac{x^2}{2} + C$

$u(x,t) = V(x,t) + w(x,t)$;

$\frac{x^2}{2}\cdot 2 + \omega_{tt} = 16(t^2-t) + \omega_{xx} + \pi tx + (t^2-t)\frac{x^2}{2} + C + w +$

$+ t\cos x - \pi tx + \frac{x^2}{2}(2-t-t^2) - 16(t^2-t+1);$

$x^2 + w_{tt} = 16(t^2 - t - t^2 + t - 1) + w_{xx} + \frac{x^2}{2}(t^2 - t) + C + w +$

$+ t\cos x + \frac{x^2}{2}(2-t-t^2) \Rightarrow -16 + w_{xx} + \frac{x^2}{2}(t^2 - t + 2 - t - t^2) +$

$+ C + w + t\cos x \Rightarrow -16\ w_{xx} + \frac{x^2}{2}(2-2t) + C + w + t\cos x;$

$C_1 = 16;$

$$\left\|\begin{array}{l} w_{tt} = 16 w_{xx} + w + \underbrace{t\cos x}_{f(x,t)\ \text{неоднорідність}} \\ w_x(0,t) = 0 \\ w_x(\pi,t) = 0 \\ w(x,0) = 2\cos x \\ w_t(x,0) = \cos 2x \end{array}\right.$$

$u(x;0) = 10 + w(x,0) = 16 + 2\cos x;$

$u_t(x,0) = \pi x - \frac{x^2}{2} + w_t(x,0) = \pi x t - \frac{x^2}{2} + \cos 2x;$

$w(x,t) = \sum_n T_n(t) X_n(x)$

$f(x,t) = \sum_n F_n(t) X_n(x)$

$T_n'' X_n = 16 X_n'' T_n + F_n X_n + X_n T_n \quad \Big| \cdot \frac{1}{X_n T_n}$

$\frac{T_n'}{T_n} = 16\frac{X_n''}{X_n} + \frac{F_n}{T_n} + 1;$

$\frac{X_n''}{X_n} = \frac{1}{16}\left[\frac{T_n''}{T_n} - 1 - \frac{F_n}{T_n}\right] = -\lambda_n^2$

$$\left\|\begin{array}{l} X_n'' + \lambda_n^2 X_n = 0 \\ X(0) = 0 \\ X'(\pi) = 0 \end{array}\right.$$

$X_n(x) = A\sin\lambda_n x + B\cos\lambda_n x;$

$X_n'(x) = A\lambda_n\cos\lambda_n x - B\lambda_n\sin\lambda_n x;$

$X'(0) = A\lambda_n = 0;$

1) $A = 0 \quad , \lambda_n \neq 0$

$X_n'(\pi) = A\lambda_n \cos\pi\lambda_n - B\lambda_n \sin\lambda_n\pi = 0$

$\lambda_n = \pi n \quad , \quad \cancel{n} \in N \; ;$

$\lambda_n = n, \quad n \in N_t \; ;$

$\{ X_n(x) = \cos \lambda x \; ; \quad \lambda = n \; , \quad n \in Z_r \}$

2) $\lambda = 0$

$X_0'' = 0$

$X = a + bx$

$X'(0) = b = 0$

$X'(\pi) = b = 0$

$X_0 = 1 \; ; \quad t\cos x = \sum F_n \cos nx \; ; \quad F_n = \begin{cases} t, & n = 1 \\ 0, & n \neq 1 \end{cases}$

$\frac{T_n''}{T_n} - 1 - \frac{F_n}{T_n} = -16\lambda^2 n T \; ;$

$T_n'' - T_n - F_n + 16\lambda_n T_n = 0$

• $n = 1$:

$T_1''\cancel{4} - T_1 - t + 16T_1 = 0$

$T_1'' + T + 16 - t$

$T_1(t) = k\sin\sqrt{15}\,t + D\cos\sqrt{15}\,t + \frac{t}{15}$

$b = \frac{1}{15}$

• $n \neq 1$

$T_n'' - T_n + 16n^2 T_n = 0$

$T_n'' + T_n(16n^2 - 1) = 0$

$T_n = C_n \sin\sqrt{16n^2 - 1} + D_n \cos(\sqrt{16n^2-1}\,t)$

І тому виходить: $T_n(t) = \{ C_n \sin\sqrt{16n^2-1}\,t + D_n\cos(\sqrt{16n^2-1}\,t) + \frac{t}{15}\delta_n \; ;$

$W(x,t) = \sum_{n=0}^{+\infty} \sin(nx)[C_n\sin(\sqrt{\cancel{\sin}\,16n^2-1}\,t) + D_n\cos(\sqrt{16n^2-1}\,t) + \frac{t}{15}\delta_n$