Домашнє завдання до практичного заняття №2
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№74

$$\begin{cases} u_t = 5u_{xx} + u_x - u \\ u_x(0,t) = 0 \\ u(1,t) = 0 \\ u(x,0) = 3 \end{cases}$$

$$u(x,t) = \sum X_n(x) T_n(t)$$

$$X_n T_n' = 5X_n'' T_n + X_n' T_n - X_n T_n \Big| \cdot \frac{1}{X_n T_n}$$

$$\frac{T_n'}{T_n} = \frac{5X_n''}{X_n} + \frac{X_n'}{X_n} - 1;$$

$$\frac{T_n'}{T_n} + 1 = 5\frac{X_n''}{X_n} + \frac{X_n'}{X_n}; \quad \frac{T_n'}{5T_n} + \frac{1}{5} = \frac{X_n''}{X_n} + \frac{X_n'}{5X_n} = -\lambda_n^2;$$

$$\begin{cases} X_n'' + \frac{1}{5}X_n' = -\lambda_n^2 X_n \\ X_n'(0) = 0 \\ X_n(1) = 0 \end{cases}$$

$$k^2 + \frac{1}{5}k + \lambda^2 = 0$$

$$D = \frac{1}{25} - 4\lambda^2$$

$$k_{1,2} = \frac{-1/5 \pm \sqrt{\frac{1}{25} - 4\lambda^2}}{2} = -\frac{1}{10} \pm \sqrt{\frac{1}{100} - \lambda^2} = -\frac{1}{10} \pm \omega i;$$

$$X_n(x) = C_1 e^{(-1/10 + \omega i)x} + C_2 e^{(-\frac{1}{10} - \omega i)x} = e^{-\frac{x}{10}}(A\sin\omega x + B\cos\omega x);$$

$$\omega^2 = \lambda^2 - \frac{1}{100};$$

$$\begin{cases} X_n'(0) = 0 = \frac{1}{10}B + A\omega \Rightarrow A = -\frac{B}{10\omega} \\ X_n(1) = 0 = e^{-1/10}(A\sin\omega + B\cos\omega) \end{cases}$$

$$\left(-\frac{B}{10\omega}\sin\omega + B\cos\omega\right)e^{-1/10} = 0;$$

$$B\left(\cos\omega - \frac{1}{10\omega}\sin\omega\right) = 0$$

$$B \neq 0; \ \cos\omega = \frac{1}{10\omega}\sin\omega; \ \operatorname{tg}\omega = 10\omega; \ \lambda_n = \frac{\omega_n}{\sqrt{\omega_n^2 + \frac{1}{100}}};$$

$X_n(x) = e^{-\frac{x}{10}} B\left(-\frac{1}{10\omega_n}\sin\omega x + \cos\omega x\right);$

$\frac{T_n'}{5T_n} + \frac{1}{5} = -\lambda_n^2 ;$

$T_n' + T_n = -5\lambda_n^2 T_n ;$

$T_n' + T_n(1+5\lambda_n^2) = 0 ;$

$T_n = C_n e^{-(1+5\lambda_n^2)} = C_n e^{-(1+5\omega_n^2+\frac{1}{20})t}$

$u(x,t) = \sum_{n=1}^{\infty} X_n(x)\, T_n(t)$

$u(x,0) = \sum_{n=1}^{\infty} C_n X_n(x) = 3$

$C_n = 2\int_0^1 3 \cdot X_n(x)dx$

№71

$u_t = 4u_{xx} + u$

$u_x(0,t) = 0$

$u_x(1,t) = 0$

$u(x;0) = 2\cos 2\pi x \cos\frac{3\pi}{2}x;$

$u(x,t) = XT';\quad XT' = 4X''T - XT' \;/\; \cdot\frac{1}{4XT}$

$\frac{T''}{4T} - \frac{1}{4} = \frac{X''}{X} = -\lambda^2;\quad X'' + \lambda^2 X = 0$

$\frac{T'}{4T} - \frac{1}{4} + \lambda^2 = 0 ;$

$X_n = A_n \sin\lambda_n x + B_n \cos\lambda_n x;$

$X_n' = A_n\lambda_n\cos\lambda_n x - B_n\lambda_n \sin\lambda_n x;$

$A_n = 0 \Rightarrow X_n' = -B_n\lambda_n \sin\lambda_n x \Rightarrow \sin\lambda_n = 0 \Rightarrow \lambda_n = \pi n,\ n\in Z$

$\{a_n = B_n\cos\lambda_n x,\ \lambda_n = \pi n,\ \pi\in Z_n\}$

$\frac{T'}{4T} + \lambda^2 - \frac{1}{4} = 0; \Rightarrow T_n = C_n e^{(1-4\lambda_n^2)t};$

$u(x,t) = \sum_{a=1}^{\infty} C_n e^{(1-4\lambda_n^2)t}\cos(\lambda_n x)$

$u(x,0) = \sum_{n=1} C_n \cos\pi n x ;\quad n\notin Z ?$

№72

$$\begin{cases} u_t = u_{xx} + u \\ u_x(0,t) = u_x(\frac{\pi}{2}, t) = 0 \\ u(x,0) = 8\cos^4 x \end{cases}$$

$u(x,t) = X(x)\,T(x)$

$XT' = X''T + XT$

$\frac{T'}{T} = \frac{X''}{X} + 1;$

$\frac{T'}{T} - 1 = \frac{X''}{X} = \lambda^2;$

$X_n = A\sin\lambda_n x + B\cos\lambda_n x;$

$X'' + \lambda^2 X = 0;$

$\frac{T'}{T} + \lambda^2 - 1 = 0,$

$X_n = A\sin\lambda_n x + B\cos\lambda_n x$

$X_n' = A_n\cos\lambda_n x - B\lambda_n\sin\lambda_n x;$

$X_n'(0) = 0; \quad X_n'(\frac{\pi}{2}) = 0; \Rightarrow \left[\begin{array}{l} \lambda_n = 0 \\ A = 0,\ D = 0,\ B \neq 0 \end{array} \right.$

$X_n' = -B\lambda_n \sin\lambda_n x;$

$\sin\lambda_n \frac{\pi}{2} = 0; \Rightarrow \lambda_n x = \pi n; \Rightarrow \lambda_n = \frac{\pi n}{\pi/2} = 2n;$

$$\begin{cases} \lambda_n = 2n \\ X_n = B_n\cos(\lambda_n x) \end{cases}$$

$\frac{T'}{T} + \lambda^2 - 1 = 0$

$T' - T + \lambda^2 T = 0$

$T_n(t) = C_n e^{t - \lambda_n^2 t}$

$u(x,t) = \sum_{n=1}^{\infty} C_n B_n e^{t - \lambda_n^2 t}\cos(\lambda_n x) = \sum_{n=1}^{\infty} C_n e^{t - \lambda_n^2 t}\cos(\lambda_n x)$

$\cos^4(x) = \frac{1}{8}(4\cos(2x) + \cos(4x) + 3) = \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x + \frac{3}{8}$

$\boxed{8\cos^4(x) = 4\cos(2x) + \cos(4x) + 3}$