N67

$$\begin{cases} X''(x) + \lambda^2 X(x) = 0 \\ X(0) + hX'(0) = 0 \\ X(l) - hX'(0) = 0 \end{cases} \qquad h > 0$$

$X(x) = A\sin\lambda x + B\cos\lambda x$

$X'(x) = A\lambda\cos\lambda x - B\lambda\sin\lambda x$

$$\begin{cases} B + hA\lambda = 0 \\ A\sin\lambda l + B\cos\lambda l - h(A\lambda\cos l - B\lambda\sin\lambda l) = 0 \end{cases}$$

$$\begin{cases} B = -A\lambda h \\ A\sin\lambda l - A\lambda h\cos\lambda l - h(A\lambda\cos\lambda l + A\lambda^2\sin\lambda l) \end{cases}$$

$A \neq 0$

$tg\,\lambda l - \lambda h - h\lambda - h^2\lambda^2\, tg\,\lambda l = 0$

$tg\, l\lambda(1 - h^2\lambda^2) = 2h\lambda$

$tg\,\lambda l = \frac{2\lambda h}{1 - h^2\lambda^2}$

Розв'язки: $\{\lambda_n\}\ n \in N$

$X_n(x) = A\sin\lambda_n x - A\lambda_n h\cos\lambda_n x = A(\sin\lambda_n x - \lambda_n h\cos\lambda_n x)$.

Домашнє робота №1

№67

$$\begin{cases} X''(x)+\lambda^2 X(x)=0 \\ X(0)+hX'(0)=0 \qquad h>0 \\ X(l)-hX'(l)=0 \end{cases}$$

$X(x)=A\sin\lambda_n+B\cos\lambda x$

$X'(x)=A\lambda\cos\lambda x-B\lambda\sin\lambda x$

$$\begin{cases} B+hA\lambda=0 \\ A\sin\lambda l+B\cos\lambda l-h(A\lambda\cos\lambda l-B\lambda\sin\lambda l)= \end{cases}$$

$=0$;

$$\begin{cases} B=-A\lambda h \\ A\sin\lambda l-A\lambda h\cos\lambda l-h(A\lambda\cos\lambda l+ \\ +A\lambda^2\sin\lambda l) \end{cases}$$;

$A\neq 0$

$\operatorname{tg}\lambda l-\lambda l-h\lambda-h^2\lambda^2\operatorname{tg}\lambda l=0$

$\operatorname{tg} l\lambda(1-h^2\lambda^2)=2h\lambda$

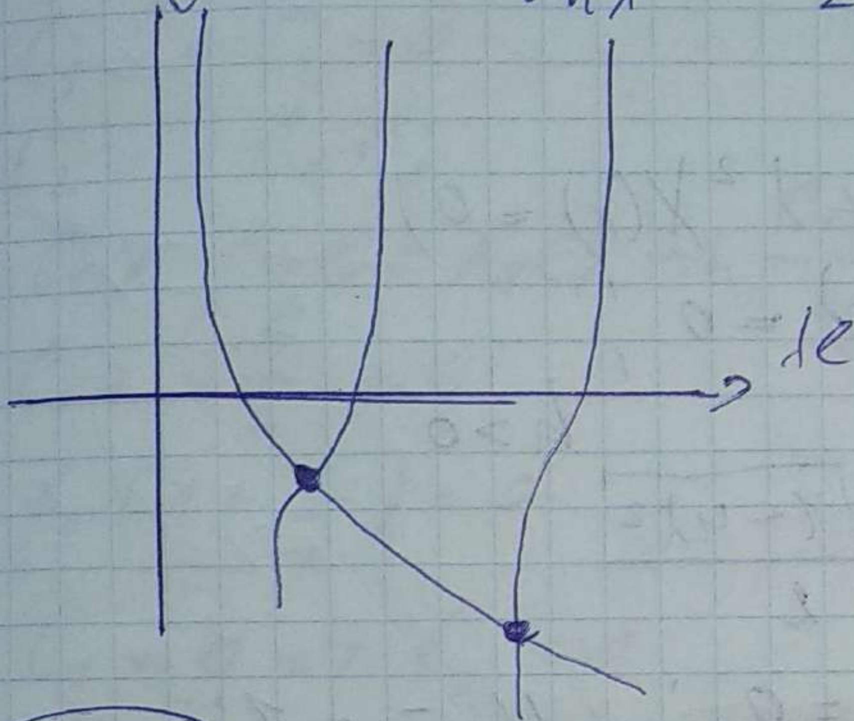
$\operatorname{tg}\lambda l=\dfrac{2\lambda h}{1-h^2\lambda^2}$;

Розв'язки: $\{\lambda_n\}$ $n\in N$;

$X_n(x)=A\sin\lambda_n x-A\lambda_n h\cos\lambda_n x=A_n(\sin\lambda_n x-$

$- \lambda_n h \cos \lambda_n x)$.

$$ctg\, \lambda l = \frac{1}{2h\lambda} - \frac{h\lambda}{2};$$



N 63

$$\begin{cases} X''(x) + \lambda^2 X(x) = 0 \\ X'(0) = X'(l) = 0 \end{cases}$$

$X'' + \lambda^2 X = 0;$

$k^2 + \lambda^2 = 0;$

$k = \pm i\lambda \Rightarrow \left[\begin{array}{l} \cos \lambda x \\ \sin \lambda x \end{array} \right.$

$X = C_1 \cos \lambda x + C_2 \sin \lambda x;$

$X' = -\lambda C_1 \sin \lambda x + \lambda C_2 \cos \lambda x;$

$\lambda C_2 = 0 \Rightarrow C_2 = 0$

$-\lambda C_1 \sin \lambda l + \lambda C_2 \cos \lambda l = 0 \Rightarrow$

$\Rightarrow -\lambda C_1 \sin \lambda l = 0;$

$\lambda_n = \frac{\pi k}{e}$, $k \in \mathbb{Z}$

$X = \cos \lambda_n x$

№66

$$\begin{cases} X''(x) + X'(x) + \lambda^2 X(x) = 0; \\ X(0) + hX'(0) = 0; \\ X(l) = 0 \qquad h > 0 \end{cases}$$

$$\mu_{1,2} = \frac{-1 \pm \sqrt{1 - 4\lambda^2}}{2}$$

1) $\lambda = 0$; $\mu_1 = 0$; $\mu_2 = -1$;

$X = C_1 + C_2 e^{-x}$;

$X' = -C_2 e^{-x}$,

$C_1 + C_2 - hC_2 = 0 \Rightarrow C_1 + C_2(1 - h) = 0$

$C_2(h - 1) = C_1$

$C_1 + C_2 e^{-l} = 0$

$C_2(h - 1) + C_2 e^{-l} \Rightarrow C_2(h - 1 + e^{-l}) = 0$;

$C_2 = 0 \Rightarrow C_1 = 0$;

2) $\lambda = \frac{1}{2}$; $\mu_1 = -\frac{1}{2}$; $\mu_2 = -\frac{1}{2}$

$X(x) = (C_1 x + C_2) e^{-x/2} = C_1 x e^{-\frac{x}{2}} + C_2 e^{-\frac{x}{2}}$,

$X' = C_1 \left(e^{-x/2} - \frac{x}{2} e^{-x/2} \right) - \frac{C_2}{2} e^{-x/2}$.

$C_2 x + C_2 + hC_1 - h\frac{C_2}{2} = 0;$

$C_1 l e^{-\frac{l}{2}} + C_2 e^{-\frac{l}{2}} = 0;$

$C_1 l + C_2 = 0;$

$C_2 = -C_1 l;$

$C_1 x - C_1 l + h_1 C_1 + h\frac{C_1 x}{2} = 0;$

$C_1 + \frac{C_1 l}{2} = 0;$

$l_1 (1 + \frac{1}{2}) = 0$

$C_1 = 0 \Rightarrow C_2 = 0$

$X = 0$

3) $\lambda < \frac{1}{2}, \ \lambda > 0; \quad \mu_{1,2} = \frac{-1 \pm i}{2};$

$X(x) = C_1 e^{(\frac{-1+i}{2})x} + C_2 e^{(\frac{-1-i}{2})x};$

$X'(x) = (\frac{-1+i}{2}) C_1 e^{(\frac{-1+i}{2})x} + C_2 (\frac{-1-i}{2}) e^{(\frac{-1-i}{2})x}.$

$C_1 + C_2 + h(\frac{-1+i}{2}) C_1 + hC_2 (\frac{-1-i}{2}) = 0$

$C_1 e^{(\frac{-1+i}{2})l} + C_2 e^{(\frac{-1-i}{2})l} = 0;$

$C_1 = -\frac{C_2 e^{(\frac{-1-i}{2})l}}{e^{(\frac{-1+i}{2})l}} = -C_2 e^{-l};$

$-C_2 e^{-l} + C_2 - h(\frac{-1+i}{2}) C_2 e^{-l} + hC_2 (\frac{-1-i}{2}) =$

$= C_2 (-e^{-l} + 1 - h(\frac{-1-i}{2}) e^{-l} + h(\frac{-1-i}{2})) = 0;$

$$C_2 = 1 \Rightarrow -e^{-\ell} + 1 + \frac{h}{2}e^{-\ell} - h\frac{i}{2}e^{-\ell} - \frac{h}{2} - h\frac{i}{2} = 0;$$

$$-e^{-\ell} + 1 + \frac{h}{2}e^{-\ell} - \frac{h}{2} = h\frac{i}{2}e^{-\ell} + h\frac{i}{2} = 0;$$

$$i\left(\frac{he^{-1}}{2} + \frac{h}{2}\right) = 1 + \frac{h}{2}e^{-\ell} - e^{-\ell} - \frac{h}{2} \Rightarrow$$

$$\Rightarrow \underline{\underline{\sqrt{1-4\lambda^2}}};$$

$$1 - 4\lambda^2 = \left(\frac{2 + he^{-\ell} - 2e^{-\ell} - h}{he^{-\ell} + h}\right)^2 \Rightarrow \lambda^2 = \frac{1}{4} \cdot \left(1 - \left(\frac{2 + he^{-1} - 2e^{-1} - h}{he^{-\ell} + h}\right)^2\right);$$

$$\lambda_n = \pm\frac{1}{2}\sqrt{1 - \left(\frac{2 + he^{-\ell} - 2e^{-\ell} - h}{he^{-\ell} + h}\right)^2}.$$