Домашняя работа до практичного заняття N6
Шамін Денис (економфізик)

N325

$\delta(\cos x)$

$\delta(y(x)) = \sum_{k=-\infty}^{\infty} [\delta(x - x_k / |y'(x_k)|]$

x_k = розв. рівняння $y(x) = 0$

$\cos x = 0$

$x_k = \frac{\pi}{2} + k\pi$, де $k = 0, \pm 1 \dots$

$|y'(x_k)| = 1$

Таким чином, $\delta(\cos x) = \sum_{k \to \infty} \delta(x - \frac{\pi}{2} + k\pi)$.

N328

$x^m p \frac{1}{x} = x^{m-1}$

$(x^m p \frac{1}{x}, \varphi) = (p \frac{1}{x}, x^m \varphi) = \int \frac{x^m \varphi(x)}{x^m} dx = \int x^{m-1} dx = (x^{m-1}, \varphi(x))$

N329

$x^n \delta(x) = 0$, де $n \in N$

нехай $\varphi \in D$, тоді

$((x^n \delta(x)), \varphi(x)) = (\delta(x), x^n \varphi(x)) = x^0 \varphi(0) = 1\varphi(0) = 1\delta(x), \varphi(x)) \Rightarrow$

$\Rightarrow 0$

N332

$\delta(x^3 - 7x + 6) = \frac{1}{4}\delta(x-1) + \frac{1}{5}\delta(x-2) + \frac{1}{20}\delta(x+3)$

$x^3 - 7x + 6 = 0$

$x_k \Rightarrow x_1 = -3$

$x_2 = 1$

$x_3 = 2$

Таким чином, $\delta(x^3 - 7x + 6) = \frac{1}{4}\delta(x-1) + \frac{1}{5}\delta(x-2) + \frac{1}{20}\delta(x+3)$

N336

$y = \begin{cases} |\sin x|, & -\pi \le x \le \pi \\ 0, & |x| \ge \pi. \end{cases}$

$f' = \{f'\} + |f|_{x_0} \delta(x - x_0)$

$|f|_{x_0} = f(x_0 + 0) - f(x_0 - 0)$

$y'(x) = \begin{cases} \cos(x), & -\pi \le x \le \pi \\ 0, & |x| \ge \pi \end{cases}$

$$\{y''(x)\} = \begin{cases} -\sin(x), & -\pi \le x \le \pi \\ 0, & |x| > \pi \end{cases}$$

$$\{y'''(x)\} = \begin{cases} -\cos(x), & -\pi \le x \le \pi \\ 0, & |x| > \pi \end{cases}$$

$$y'''(x) = -\cos\,\tau'(x) + 0$$

$$y^{(4)}(x) = -\cos\,\tau''(x)$$

$$y^{(n)}(x) = -\cos\,\tau^{(n-3)}(x) \quad n > 3$$

N343

$$u(x) = x^2\tau'(x)$$

$$(u(x),\varphi) = (x^2\tau',\varphi) = (\tau', x^2\varphi) = -(\tau, (x^2\varphi)') = -(\tau, \varphi + 2x\varphi)$$

$$= -(\tau,\varphi) - (\tau, 2x\varphi') = -\varphi(0) - 2x\varphi'\Big|_{x=0} = -\varphi(0) = (-\tau,\varphi).$$

N344

$$(u(x)) = x^2\tau''(x).$$

$$(u(x),\varphi) = (x^2\tau'',\varphi) = (\tau, (2x^2\varphi'')) = (\tau, (\varphi + x^2\varphi')') =$$

$$= (\tau, 2\varphi' + 2x\varphi') = 2(\tau,\varphi') + 2(\tau, x\varphi') = 2\varphi'(0) + 2x\varphi'(x)\Big|_{x=0} =$$

$$= 2\varphi'(0) + 2\varphi'(x) = (2\tau'(x), 2\varphi(x)).$$