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Д/з №4
№188

$\Delta u(x,y) = 0$
$u(0,y) = V_1$
$u(a,y) = V_2$
$u(x,0) = 0$
$u(x,b) = 0$

I

$\Delta u(x,y) = 0$
$u(a,0) = 0$
$u(x,b) = 0$
$u(0,y) = V_1$
$u(a,y) = 0$

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$u(0,y) = V_1$
$u(a,y) = 0$

$u(x,y) = \sum X_n(x) Y_n(y)$
$X_n'' Y_n + X_n Y_n'' = 0;$
$\frac{X_n''}{X_n} = -\frac{Y_n''}{Y_n} = \lambda_n^2;$

II

$\Delta u(x,y) = 0$
$u(x,0) = 0$
$u(x,b) = 0$
$u(0,y) = 0$
$u(a,y) = V_2$

$X_n'' - \lambda_n^2 X_n$
$X(0) = V_1$
$X(a) = 0$

$k^2 - \lambda^2 = 0$
$k = \pm\lambda$
$X_n(x) = C_{1n} e^{\lambda_n x} + C_{2n} e^{-\lambda_n x};$

$u(x,y) = \sum_{n=1}^{\infty} \sin\lambda_n y (C_{1n} e^{\lambda_n x} + C_{2n} e^{-\lambda_n x});$

$u(a,y) = 0 = \sum_{n=1}^{\infty} \sin\lambda_n y (C_{1n} e^{\lambda_n x} + C_{2n} e^{-\lambda_n x}).$

$C_{1n} e^{\lambda_n x} + C_{2n} e^{-\lambda_n x} = 0$ $\Rightarrow$

$\Rightarrow$ $u(0,y) = \sum_{n=1}^{\infty} \sin\lambda_n y (C_{1n} - C_{2n} e^{2\lambda_n x}) = V_1$

$\sum C_{1n} \sin\lambda_n y (1 - e^{2\cdot\lambda_n}) = V_1$

$$C_{1n} = \frac{U_1}{1 - e^{2\pi n a}};$$

$$U^{I}(x,y) = \sum_{n=1}^{\infty} \sin\frac{\pi n y}{b} \frac{U_1}{(1 - e^{2\pi n x})} \left(e^{\pi x} - e^{\frac{2\pi a n}{b} x}\right).$$

N199

$$\begin{cases} \Delta U = 0 \\ U_\rho(R,\varphi) = A_0 + B\cos^2\varphi \end{cases}$$

$$\Delta U = U_{\rho\rho} + \frac{1}{\rho}U_\rho + \frac{1}{\rho^2}U_{\varphi\varphi} = 0$$

$$U^{int}(\rho) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(\frac{\rho}{R}\right)^n (A_n\cos(n\varphi) - B_n\sin(n\varphi));$$

$$U^{ext}(\rho) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(\frac{R}{\rho}\right)^n (A_n\cos(n\varphi) + B_n\sin(n\varphi));$$

$$U_0 + U_1\cos n\varphi = U_\rho;$$

$$U_\rho^{int} = \sum_{n=1}^{\infty}\frac{n\rho^{n-1}}{R^n}(A_n\cos(n\varphi) + B_n\sin(n\varphi))$$

$$A + B\cos^2\varphi = \frac{A_0}{2} + \sum_{n=1}^{\infty}(A_n\cos(n\varphi) + B_n\sin(n\varphi))$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} f(\varphi)\cos n\varphi\, d\varphi;$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} f(\varphi)\sin n\varphi\, d\varphi;$$

$$\cos^3\varphi = \frac{1}{4}(3\cos\varphi + \cos 3\varphi) = \frac{3}{4}\cos\varphi + \frac{1}{4}\cos 3\varphi;$$

$$n = 1 \Rightarrow A_1 = \frac{3}{4};\ B_1 = 0$$

$$n = 3 \Rightarrow A_3 = \frac{1}{4};\ B_3 = 0$$

$$A_n = 0;\ B_n = 0;\quad A_0 = 2A$$

$$U^{int}(\rho,\varphi) = A + \left(\frac{\rho}{R}\right)\frac{3}{4}\cos\varphi + \left(\frac{\rho}{R}\right)^3\frac{1}{4}\cos 3\varphi;$$

$$U^{ext}(\rho,\varphi) = A + \left(\frac{R}{\rho}\right)\frac{3}{4}\cos\varphi + \left(\frac{\rho}{R}\right)^3\frac{1}{4}\cos 3\varphi.$$

№224

$$\begin{cases} \Delta u = 0 \\ u(x,y) = x^2 - y^2 \end{cases}$$

$x^2 + y^2 = R^2$

$\rho = R$; $\rho^2\cos^2\varphi - \rho^2\sin^2\varphi = 0 = \rho^2\cos 2\varphi = R^2\cos 2\varphi$

$u(R,\varphi) = R^2\cos 2\varphi$

$R^2\cos 2\varphi = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n\cos n\varphi + B_n\sin n\varphi$;

$A_n = 0$

$n = 2 \Rightarrow A_n = 1$; $B_n = 0$

$u^{int}(\rho,\varphi) = \left(\frac{\rho}{R}\right)^2 \cos 2\varphi$;

$u^{ext}(\rho,\varphi) = \left(\frac{R}{\rho}\right)^2 \cos 2\varphi$;