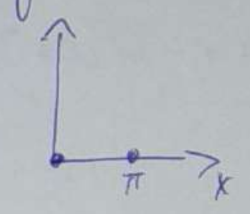
Домашнє до практичного заняття N4
Шалій Денис 98, 113, 141, 175

N98

$$\begin{cases} u_t = u_{xx} + u + 2\cos 3x \\ u_x(0,t) = u_x(\pi,t) = 0 \\ u(x,0) = \cos x \end{cases}$$

$u(x,t) = v(x,t) + \omega(x,t)$

$$\begin{cases} v(0,t) = 0 \\ v(\pi,t) = 0; \end{cases}$$



$v = \pi.$

$u(x,t) = \pi + \omega(x,t)$

$\omega_t = \omega_{xx} + \pi + \omega + 2\cos 3x$

N141

$u_{tt} = a^2 u_{xx}$

$u(0,t) = u_x(l,t) = 0$

$u(x,0) = 1$

$u_t(x,0) = \sin\frac{\pi x}{2l}$

$u(x,t) = \sum T_n(t) X_n(x)$

$T''X = a^2 X'' T$

$\frac{T''}{a^2 T} = \frac{X''}{X} = -\lambda^2;$

$X''_n + \lambda^2 X_n = 0;$

$X_n = A_n \sin\lambda_n x + B_n \cos\lambda_n x \xrightarrow{u(0,t)} B_n = 0$

$X'_n = \lambda_n A_n \cos\lambda_n x - B_n\lambda_n \sin\lambda_n x \xrightarrow{u(l,t)} A_n\lambda_n\cos\lambda_n l = 0 \Rightarrow \cos\lambda_n l = 0$

$$\begin{cases} \lambda_n = \frac{\pi}{2l}(2n-1) \\ X_n = A_n \sin\lambda_n x; \end{cases} \quad n \in Z$$

$T'' + a^2\lambda^2 T = 0$

$T_n = \tilde{A}_n \cos(a,\lambda,t) + \tilde{B}_n \sin(a,\lambda_n,t);$

$T_n' = B_n\lambda_n\cos(a,\lambda,t) - A_n\lambda_n\sin(a,\lambda_n,t);$

$T_n(0) = 1 \Rightarrow \tilde{A}_n = 1;$

$T_n'(0) = 1 \Rightarrow \tilde{B}_n = 1;$

$u(x,t) = \sum_{n=1}^{\infty} A_n$ ~~...~~ $T_n X_n = \sum_n A_n \sin\lambda_n x(\sin(a,\lambda,t) = \sum_n A_n \cdot$
$\cdot \sin(\lambda,n,x)\sin(a,\lambda_n,t)$

$u_t(x,t) = A_n a \lambda_n \sin\lambda_n x \cos(0) = \sin\frac{\pi x}{2l}$;

$\lambda_n = \frac{\pi}{2l}(2n-1)$

$A_n = \frac{2l}{\pi}$

$u(x,t) = \frac{2l}{\pi a}\sin\left(\frac{\pi x}{2l}\right)\sin\left(\frac{\pi a}{2l}t\right)$

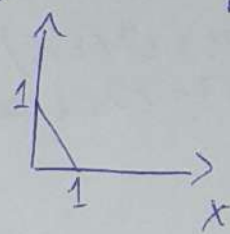
N113

$u_t = u_{xx} + u - x + 1 - u\cos^3\frac{5\pi x}{2}$

$u_x = (0,t) = 1$

$u(1,t) = 0$

$u(x,0) = x - 1$



$\vartheta = -x + 1$

$u(x,t) = v(x,t) + \omega(x,t)$

$v(0,t) = 1$

$v(1,t) = 0$

$u(x,t) = \vartheta(x,t) + \omega(x,t) = 1 - x + \omega(x,t)$

$\omega_t = \omega_{xx} + 1 - x + \omega - x + 1 + (1 - x + \omega)\cos^3\frac{5\pi x}{2}$

N175

$u_{tt} = 16u_{xx} + u + t\cos x - \pi t x + \frac{x^2}{2}(2t - t^2) - 16(t^2 - t + 1)$

$u_x(0,t) = \pi t$

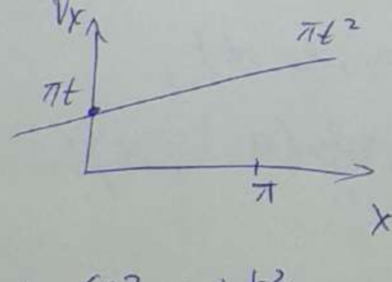
$u_x(\pi,t) = \pi t^2$

$u(x,0) = 16 + 2\cos x$

$u_t(x,0) = \pi x - \frac{x^2}{2} + \cos 2x$

$v_x(0,t) = \pi t$

$v_x(\pi,t) = \pi t^2$



$v_x(x,t) = a + bx$

$v_x(0,t) = \pi t = a$;

$v_x(\pi,t) = \pi t^2 = a + b\pi$;

$b = t^2 - t$

$v_x = \pi t + (t^2 - t)x$.

$v(x,t) = \pi t x + (t^2 - t)\frac{x^2}{2} + C$

$u(x,t) = v(x,t) + \omega(x,t)$;

$$\frac{x^2}{2}\cdot 2 + w_{tt} = 16(t^2-t) + w_{xx} + \pi t x + (t^2-t)\frac{x^2}{2} + C + w +$$

$$+ t\cos x - \pi t x + \frac{x^2}{2}(2-t-t^2) - 16(t^2-t+1);$$

$$x^2 + w_{tt} = 16(t^2-t-t^2+t-1) + w_{xx} + \frac{x^2}{2}(t^2-t) + C + w +$$

$$+ t\cos x + \frac{x^2}{2}(2-t-t^2) \Rightarrow -16 + w_{xx} + \frac{x^2}{2}(t^2-t+2-t-t^2) +$$

$$+ C + w + t\cos x \Rightarrow -16\; w_{xx} + \frac{x^2}{2}(2-2t) + C + w + t\cos x;$$

$C_1 = 16;$

$$\begin{cases} w_{tt} = 16 w_{xx} + w + \underbrace{t\cos x}_{f(x,t)\text{ неоднорідність}} \\ w_x(0,t) = 0 \\ w_x(\pi,t) = 0 \\ w(x,0) = 2\cos x \\ w_t(x,0) = \cos 2x \end{cases}$$

$$u(x,0) = 10 + w(x,0) = 16 + 2\cos x;$$

$$u_t(x,0) = \pi x - \frac{x^2}{2} + w_t(x,0) = \pi x t - \frac{x^2}{2} + \cos 2x;$$

$$w(x,t) = \sum_n T_n(t) X_n(x)$$

$$f(x,t) = \sum_n F_n(t) X_n(x)$$

$$T_n'' X_n = 16 X_n'' T_n + F_n X_n + X_n T_n \;\Big|\cdot \frac{1}{X_n T_n}$$

$$\frac{T_n''}{T_n} = 16\frac{X_n''}{X_n} + \frac{F_n}{T_n} + 1;$$

$$\frac{X_n''}{X_n} = \frac{1}{16}\left[\frac{T_n''}{T_n} - 1 - \frac{F_n}{T_n}\right] = -\lambda_n^2$$

$$\begin{cases} X_n'' + \lambda_n^2 X_n = 0 \\ X(0) = 0 \\ X'(\pi) = 0 \end{cases}$$

$$X_n(x) = A\sin\lambda_n x + B\cos\lambda_n x;$$

$$X_n'(x) = A\lambda_n\cos\lambda_n x - B\lambda_n\sin\lambda_n x;$$

$$X'(0) = A\lambda_n = 0;$$

1) $A = 0$, $\lambda_n \neq 0$

$X_n'(\pi) = A\lambda_n \cos\pi\lambda_n - B\lambda_n \sin\lambda_n\pi = 0$

$\lambda_n = \pi n$, $n \in N$;

$\lambda_n = n$, $n \in N$;

$\{ X_n(x) = \cos\lambda x$; $\lambda = n$, $n \in Z_+ \}$

2) $\lambda = 0$

$X_0'' = 0$

$X = a + bx$

$X'(0) = b = 0$

$X'(\pi) = b = 0$

$X_0 = 1$; $t\cos x = \sum F_n \cos nx$; $F_n = \begin{cases} t, & n = 1 \\ 0, & n \neq 1 \end{cases}$

$\frac{T_n''}{T_n} - 1 - \frac{F_n}{T_n} = -16\lambda^2 n T$;

$T_n'' - T_n - F_n + 16\lambda_n T_n = 0$

• $n = 1$:

$T_1'' - T_1 - t + 16T_1 = 0$

$T_1'' + T + 16 = t$

$T_1(t) = K\sin\sqrt{15}\,t + D\cos\sqrt{15}\,t + \frac{t}{15}$

$b = \frac{1}{15}$

• $n \neq 1$

$T_n'' - T_n + 16n^2 T_n = 0$

$T_n'' + T_n(16n^2 - 1) = 0$

$T_n = C_n \sin\sqrt{16n^2 - 1} + D_n \cos(\sqrt{16n^2-1}\,t)$

З тому виходить: $T_n(t) = \{ C_n \sin\sqrt{16n^2-1}\,t + D_n\cos(\sqrt{16n^2-1}\,t) +$

$+ \frac{t}{15}\delta_n$;

$W(x,t) = \sum_{n=0}^{+\infty} \sin(nx)[C_n \sin(\sqrt{16n^2-1}\,t) + D_n\cos(\sqrt{16n^2-1}\,t) + \frac{t}{15}\delta_n$

$$W(x,0) = 2\cos x = \sum_n \cos(nx)\, \Phi_n;$$

$$\Phi_n = \begin{cases} 2, & n=1 \\ 0, & n \neq 1 \end{cases} = 2\delta_{n1}$$

$$W_t(x,0) = \cos 2x = \sum \cos(nx) \Big[C_n \sqrt{16n^2-1} \cos\left(\sqrt{16n^2-1}\, t\right) -$$

$$- \Phi_n \left(\sqrt{16n^2-1}\, t + \frac{f_n}{15} \right) = \sum_{n=0}^{\infty} \cos nx \left[C_n \sqrt{16n^2-1} + \frac{f_n}{15} \right] =$$

$$= \cos 2x;$$

$n = 2$

$$C_2 \sqrt{63} = 1 \Rightarrow C_2 = \frac{1}{\sqrt{63}}$$

$n = 1$

$$C_1 \sqrt{15} + \frac{1}{15} = 0 \Rightarrow C_1 = \frac{-1}{15\sqrt{15}}$$

$$C_n = \frac{\delta_{n2}}{\sqrt{63}} + \frac{\delta_{n1}}{15\sqrt{15}}$$

$$W(x,t) = \cos x \left[\frac{-1}{15\sqrt{15}} \sin(\sqrt{15}\, t) + 2\cos\sqrt{15}\, t + \frac{t}{15} \right] +$$

$$+ \cos 2x \left[\frac{1}{\sqrt{63}} \sin(\sqrt{63}\, t) \right]$$

$$U(x,t) = V(x,t) + W(x,t) = \pi t x (t^2 - t) \frac{x^2}{3} + 16 + \frac{\cos 2x}{\sqrt{63}} \cdot$$

$$\cdot \sin\sqrt{63}\, t + \cos x \left[2\cos\sqrt{15}\, t - \frac{\sin\sqrt{15}\, t}{15\sqrt{15}} + \frac{1}{15} \right].$$