

Составить задание по фактурному занятию 1/2
Машин Денис

(N 74)

$$U_t = 5U_{xx} + U_x - U$$

$$U_x(0, t) = 0$$

$$U(1, t) = 0$$

$$U(x, 0) = 3$$

$$U(x, t) = \sum X_n(x) T_n(t)$$

$$X_n T_n' = 5 X_n'' T_n + X_n' T_n - X_n T_n \quad \cdot \frac{1}{X_n T_n}$$

$$\frac{T_n'}{T_n} = \frac{5 X_n''}{X_n} + \frac{X_n'}{X_n} - 1;$$

$$\frac{T_n'}{T_n} + 1 = \frac{5 X_n''}{X_n} + \frac{X_n'}{X_n}; \quad \frac{T_n'}{5 T_n} + \frac{1}{5} = \frac{X_n''}{X_n} + \frac{X_n'}{5 X_n} = -\lambda_n^2;$$

$$X_n'' + \frac{1}{5} X_n' = -\lambda_n^2 X_n$$

$$X_n'(0) = 0$$

$$X_n(1) = 0$$

$$k^2 + \frac{1}{5}k + \lambda^2 = 0$$

$$D = \frac{1}{25} - 4\lambda^2$$

$$k_{1,2} = \frac{-1/5 \pm \sqrt{\frac{1}{25} - 4\lambda^2}}{2} = -\frac{1}{10} \pm \sqrt{\frac{1}{100} - \lambda^2} = -\frac{1}{10} \pm \omega i;$$

$$X_n(x) = C_1 e^{(-\frac{1}{10} + \omega i)x} + C_2 e^{(-\frac{1}{10} - \omega i)x} = e^{-\frac{x}{10}} (A \sin \omega x + B \cos \omega x);$$

$$\omega^2 = \lambda^2 - \frac{1}{100};$$

$$\begin{cases} X_n'(0) = 0 = \frac{1}{10} B + A \omega \Rightarrow A = -\frac{B}{10 \omega} \\ X_n(1) = 0 = e^{-\frac{1}{10}} (A \sin \omega + B \cos \omega) \end{cases}$$

$$(-\frac{B}{10 \omega} \sin \omega + B \cos \omega) e^{-\frac{1}{10}} = 0;$$

$$B (\cos \omega - \frac{1}{10 \omega} \sin \omega) = 0$$

$$B \neq 0; \cos \omega = \frac{1}{10 \omega} \sin \omega; \operatorname{tg} \omega = 10 \omega; \lambda_n = \frac{\omega_n}{\sqrt{\omega_n^2 + \frac{1}{100}}};$$

$$X_n(x) = e^{-\frac{x}{10}} B\left(-\frac{1}{10\omega_n} \sinh \omega_n x + \cosh \omega_n x\right);$$

$$\frac{T_n'}{5T_n} + \frac{1}{5} = -\lambda_n^2;$$

$$T_n' + T_n = -5\lambda_n^2 T_n;$$

$$T_n' + T_n(1 + 5\lambda_n^2) = 0;$$

$$T_n = C_n e^{-(1+5\lambda_n^2)t} = C_n e^{-(1+5\omega_n^2 + \frac{1}{20})t}$$

$$U(x,t) = \sum_{n=1}^{\infty} X_n(x) T_n(t)$$

$$U(x,0) = \sum_{n=1}^{\infty} C_n X_n(x) = 3$$

$$C_n = 2 \int_0^1 3 \cdot X_n(x) dx$$

(N71)

$$U_t = 4U_{xx} + U$$

$$U_x(0,t) = 0$$

$$U_x(1,t) = 0$$

$$U(x,0) = 2 \cos 2\pi x \cos \frac{3\pi}{2} x;$$

$$U(x,t) = XT'; \quad XT' = 4X''T - XT' \quad | \cdot \frac{1}{4XT}$$

$$\frac{T''}{4T} - \frac{1}{4} = \frac{X''}{X} = -\lambda^2; \quad X'' + \lambda^2 X = 0$$

$$\frac{T'}{4T} - \frac{1}{4} + \lambda^2 = 0;$$

$$X_n = A_n \sinh \lambda_n x + B_n \cosh \lambda_n x;$$

$$X_n' = A_n \lambda_n \cosh \lambda_n x - B_n \lambda_n \sinh \lambda_n x;$$

$$A_n = 0 \Rightarrow X_n' = -B_n \lambda_n \sinh \lambda_n x \Rightarrow \sinh \lambda_n = 0 \Rightarrow \lambda_n = \pi n, \quad n \in \mathbb{Z}$$

$$\{a_n = B_n \cos \lambda_n x, \quad \lambda_n = \pi n, \quad n \in \mathbb{Z}\}$$

$$\left\{ \frac{T'}{4T} + \lambda^2 - \frac{1}{4} = 0 \right\} \Rightarrow T_n = C_n e^{(1-4\lambda_n^2)t};$$

$$U(x,t) = \sum_{n=-\infty}^{\infty} C_n e^{(1-4\lambda_n^2)t} \cos(\lambda_n x)$$

$$U(x,0) = \sum_{n=-\infty}^{\infty} C_n \cos \pi n x; \quad n \in \mathbb{Z}?$$

N72

$$\begin{cases} u_t = u_{xx} + u \\ u_x(0, t) = u_x(\frac{\pi}{2}, t) = 0 \\ u(x, 0) = 8 \cos^4 x \end{cases}$$

$$X'' + \lambda^2 X = 0;$$

$$\frac{T'}{T} + \lambda^2 - 1 = 0,$$

$$X_n = A \sin \lambda_n x + B \cos \lambda_n x$$

$$X_n' = A \cos \lambda_n x - B \lambda_n \sin \lambda_n x;$$

$$X_n'(0) = 0; \quad X_n'(\frac{\pi}{2}) = 0; \Rightarrow \begin{cases} \lambda_n = 0 \\ B = 0, D = 0, B \neq 0 \end{cases}$$

$$X_n' = -B \lambda_n \sin \lambda_n x;$$

$$\sin \lambda_n \frac{\pi}{2} = 0; \Rightarrow \lambda_n x = \pi n; \Rightarrow \lambda_n = \frac{\pi n}{\pi/2} = 2n;$$

$$\begin{cases} \lambda_n = 2n \\ X_n = B_n \cos(\lambda_n x) \end{cases}$$

$$\frac{T'}{T} + \lambda^2 - 1 = 0$$

$$T' - T + \lambda^2 T = 0$$

$$T_n(t) = C_n e^{t - \lambda_n^2 t}$$

$$u(x, t) = \sum_{n=1}^{\infty} C_n B_n e^{t - \lambda_n^2 t} \cos(\lambda_n x) = \sum_{n=1}^{\infty} C_n e^{t - \lambda_n^2 t} \cos(\lambda_n x)$$

$$\cos^4(x) = \frac{1}{8} (4 \cos(2x) + \cos(4x) + 3) = \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x + \frac{3}{8}$$

$$8 \cos^4(x) = 4 \cos(2x) + \cos(4x) + 3$$

$$u(x, t) = X(x) T(t)$$

$$X T' = X'' T + X T$$

$$\frac{T'}{T} = \frac{X'}{X} + 1;$$

$$\frac{T'}{T} - 1 = \frac{X''}{X} = -\lambda^2;$$

$$X_n = A \sin \lambda_n x + B \cos \lambda_n x;$$