№67

$$\begin{cases} X''(x) + \lambda^2 X(x) = 0 \\ X(0) + hX'(0) = 0 \\ X(\ell) - hX'(\ell) = 0 \end{cases} \qquad h > 0$$

$X(x) = A\sin \lambda x + B\cos \lambda x$

$X'(x) = A\lambda\cos \lambda x - B\lambda \sin \lambda x$

$$\begin{cases} B + hA\lambda = 0 \\ A\sin\lambda\ell + B\cos\lambda\ell - h(A\lambda\cos\lambda\ell - B\lambda\sin\lambda\ell) = 0 \end{cases}$$

$$\begin{cases} B = -A\lambda h \\ A\sin\lambda\ell - A\lambda h\cos\lambda\ell - h(A\lambda\cos\lambda\ell + A\lambda^2\sin\lambda\ell) \end{cases}$$

$A \neq 0$

$\operatorname{tg}\lambda\ell - \lambda h - h\lambda - h^2\lambda^2\operatorname{tg}\lambda\ell = 0$

$\operatorname{tg}\ell\lambda(1 - h^2\lambda^2) = 2h\lambda$

$\operatorname{tg}\lambda\ell = \dfrac{2\lambda h}{1 - h^2\lambda^2}$

$\operatorname{ctg}\lambda\ell = \dfrac{1}{2h\lambda} - \dfrac{h\lambda}{2}$

Pozbierajmy: $\{\lambda_n\}_{n\in\mathbb{N}}$

$X_n(x) = A\sin\lambda_n x - A\lambda_n h\cos\lambda_n x = A_n(\sin\lambda_n x - \lambda_n h\cos\lambda_n x)$.