of a compressed quasiatom or in its self-compression without external pressure? Such a question was examined for the first time by Salpeter in Refs. 13, 14 while solving the problem of stability of cosmological objects (black holes, white dwarfs, and neutron stars).

As shown in Refs. 13–16, the presence of the electron-nuclear attraction in a preliminarily compressed degenerate nonrelativistic electron gas results in the Fermi reduction of the electron gas pressure by several percents, which is not sufficient, obviously, for the plasma stabilization and, what is more, for self-compression. For the sake of justice, it is necessary to mention Ref. 15, where an estimation of the role of the Coulomb interaction for a compressed relativistic degenerate gas was performed with the same negative result. However, one should not consider this estimation to be valid because the authors used the nonrelativistic expression for Coulomb interaction energy $U = -e\varphi(r) = -Ze^2/r$ in the calculations based on the equation of state of the relativistic degenerate gas. This problem will be examined in more details below.

On the other hand, the detailed analysis of the interaction of a nucleus with a relativistic degenerate gas without preliminary external compression in Ref. 4 allows one to conclude that, in a nucleus with $Z \ll 1600$, the presence of the electron environment corresponds to an ordinary (not compressed) atom.

Thus, the problem of evolution of a quasiatom preliminarily compressed by external forces up to the state of a relativistic degenerate gas seems to be unexplored. The further analysis shows that this very system is capable under certain conditions to transfer to the state of self-organizing electron-nuclear collapse whose evolution facilitates the formation of superheavy nuclei.

2.2 The Energy of Interaction of a Compressed Relativistic Degenerate Gas of Electrons with a Nucleus in the Volume of a Neutral Wigner–Seitz Cell

Let's calculate the energy of interaction between all particles in the volume of a neutral Wigner–Seitz elementary cell constructed near one of nuclei.

At first, we calculate the energy of interaction of one electron with a nucleus and other electrons. In general, the movement of an electron in a field with potential $\varphi(r)$ is described by the Dirac equation

$$\{E + e\varphi(r) - \hat{\beta}m_e c^2 - \hat{\boldsymbol{\alpha}}c\mathbf{p}\}u = 0, \tag{3}$$

$$\hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{1} & 0 \\ 0 & -\hat{1} \end{pmatrix}, \hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix}, \tag{4}$$

for the four-component wave function u.

Here, $\hat{\beta}$ and $\hat{\boldsymbol{\alpha}}$ — scalar and vectorial Dirac matrices determined through the four-component identity matrix $\hat{1}$ and the four-component vectorial Pauli matrix $\hat{\boldsymbol{\sigma}}$.

If we multiply this equation by the operator $\{E + e\varphi(r) - \hat{\beta}m_e c^2 - \hat{\boldsymbol{\alpha}}c\hat{\mathbf{p}}\}$ and use the operator identities

$$(\hat{\boldsymbol{\sigma}}\mathbf{A})(\hat{\boldsymbol{\sigma}}\mathbf{B}) = (\mathbf{AB}) + i\hat{\boldsymbol{\sigma}}[\mathbf{AB}],\ (\hat{\boldsymbol{\sigma}}\mathbf{A})(\hat{\boldsymbol{\sigma}}\mathbf{A}) = \mathbf{A}^2 \tag{5}$$

and also the permutation relations for components of the vector Dirac matrix

$$\hat{\alpha}_i\hat{\alpha}_k + \hat{\alpha}_k\hat{\alpha}_i = 2\hat{\delta}_{ik}, \tag{6}$$

then we get a differential equation of the second order like the Schrödinger equation:

$$\{\hat{\mathbf{p}}^2/2m_e + U_{\text{eff}}\}u = \varepsilon u, \tag{7}$$

$$\varepsilon = [E^2 - (m_e c^2)^2]/2m_e c^2, \tag{8}$$

$$U_{\text{eff}} \equiv U_{\text{eff}}(r, E) = -e\varphi(r)(E/m_e c^2) - [e\varphi(r)]^2/2mc^2. \tag{9}$$

Here, $U_{\text{eff}}(r, E)$ — the effective potential energy determining the movement of an electron with full energy E in the field with potential $\varphi(r)$.

Eq. 9 for the effective potential energy $U_{\text{eff}}(r, E)$ differs essentially from the "traditional" expression for interaction energy, $U_0(r) = -e\varphi(r)$, which is valid only in the nonrelativistic case (at $E \approx m_e c^2$ and $(E - m_e c^2)/m_e c^2 \ll 1$) and for comparatively weak fields ($|e\varphi(r)| \ll m_e c^2$).

In particular, for nonrelativistic particles with mass m and full energy $E \approx mc^2$ ($(E - mc^2)/mc^2 \ll 1$), the effective potential energy in a very strong field is determined by the first two terms in Eq. 9 and looks like

$$U_{\text{eff}} \approx U_{\text{eff}}(r, mc^2) \approx -e\varphi(r) - [e\varphi(r)]^2/2mc^2. \tag{10}$$

It follows from Eq. 10 that the effect of a strong scalar potential (at $|e\varphi(r)| > 2mc^2$) for nonrelativistic charged particles corresponds always to a nonlinear attraction ($U_{\text{eff}}(r, mc^2) < 0$) irrespective of the sign of a particle charge. This effect was discussed above in the analysis of the interaction of condensed pions with protons in a nucleus according to the Migdal model (see Ref. 4).

In the case of relativistic charged particles, except for a nonlinear attraction, the effective scalar potential can be increased due to the Lorentz transformation of a field with scalar potential $\varphi(r)$ for fast moving particles.

In view of the full energy of one relativistic electron

$$\begin{aligned} E &= -e\varphi(r) + (p^2c^2 + m_e^2c^4)^{1/2} \\ &= -e\varphi(r) + T_1(p) + m_e c^2 \\ &\equiv -e\varphi(r) + \gamma_p m_e c^2 \end{aligned} \tag{11}$$

relation Eq. 9 allows one to get expressions for both the effective potential energy $U_{\text{eff}}(r, p)$ for an electron with momentum p and the similar effective energy of an electron $U_{\text{eff}}(r, p_F)$ on the surface of a Fermi sphere (with the limit momentum p_F)

$$\begin{aligned} U_{\text{eff}}(r, p) &= -e\varphi(r) - e\varphi(r)[T_1(p) - e\varphi(r)/2]/m_e c^2 \\ &= -e\varphi(r)\gamma_p + [e\varphi(r)]^2/2m_e c^2, \end{aligned} \tag{12}$$

$$\begin{aligned} U_{\text{eff}}(r, p_F) &= -e\varphi(r)E_F/m_e c^2 + [e\varphi(r)]^2/2m_e c^2 \\ &= -e\varphi(r)\gamma_F + [e\varphi(r)]^2/2m_e c^2. \end{aligned} \tag{13}$$

Here, $E_F = (p_F^2c^2 + m_e^2c^4)^{1/2} = \gamma_F m_e c^2$ — Fermi energy, $T_1(p) = (p^2c^2 + m_e^2c^4)^{1/2} - m_e c^2 = (\gamma_p - 1)m_e c^2$ — kinetic energy of one electron with momentum p in the degenerate relativistic electron gas, γ_p and γ_F are, respectively, the relativistic Lorentz factors for an electron with momentum p and maximal momentum p_F on the surface of the Fermi sphere.

Let's note a basic difference of the expressions for the effective potential energy of an electron in the examined quasiatom compressed by external pressure and one in an ordinary atom held by the internal Coulomb interaction only.

In a bound electron-nuclear system without external influence (without additional external pressure), the full energy $E = -e\varphi(r) + T_1(p) + m_e c^2$ is always less than the rest energy $m_e c^2$. This follows, obviously, from the fact that the steady and bound state of an electron in an atom is impossible at $-e\varphi(r) + T_1(p) > 0$. Using the condition $E < m_e c^2$, it can be found from Eq. 10 that the maximal (by absolute value) potential energy of an electron in an ordinary atom is always limited

$$|U_{\text{eff(atom)}}(r, p_F)|_{(\max)} \leq e\varphi(r) + [e\varphi(r)]^2/2m_e c^2. \tag{14}$$

In the case of a compressed quasiatom, the full energy E of a single electron depends on external pressure which keeps the compressed quasiatom from dispersion and may increase its kinetic energy $T_1(p) = (\gamma_p - 1)m_e c^2$ unlimitedly, according to the unlimited increase of the ratio $E/m_e c^2$. It should be noted that an increase in the kinetic energy $\langle T_1(p)\rangle \gg \langle e\varphi(r)\rangle$ is possible under compression, which is unattainable, in principle, in the Coulomb field for which $\langle T_1(p)\rangle = \langle e\varphi(r)\rangle/2$ by the virial theorem.

Such a compression results in a drastic increase in the effective potential energy of an electron in the compressed quasiatom

$$|U_{\text{eff(quasiatom)}}(r, p)|_{(\max)} > e\varphi(r) + [e\varphi(r)]^2/2m_e c^2. \tag{15}$$

In particular, if $E \gg \langle e\varphi(r)\rangle$ then

$$|U_{\text{eff(quasiatom)}}(r, p)|_{(\max)} \gg e\varphi(r) + [e\varphi(r)]^2/2m_e c^2, \tag{16}$$

which corresponds to a very significant enhancement of the interaction of an electron with a nucleus. The physical reason for this is related to a relativistic transformation of a scalar potential in the case of moving particles.

This very fact allows us to realize the effect of self-compression of the electron-nuclear plasma considered below which may occur at a preliminary threshold compression of the substance. For "ordinary" atoms not compressed by external forces up to the state of degenerate relativistic gas, such an effect is impossible in principle.

Let's note one more important fact.

In the presence of other electrons in the volume of a Wigner–Seitz cell, the potential $\varphi(r)$ (and, consequently, effective potential U_{eff}) should be determined via the nucleus potential $\varphi_0(r) = Ze/r$ and the average potential formed by other electrons. The final concentration of electrons $n_e(r) = (1/3\pi^2\hbar^3c^3)[E_F - U_{\text{eff}}(r, p_F)]^3$ in the volume of a Wigner–Seitz cell is a result of such a self-coordinated interaction.

This problem may be solved by using the relativistic Thomas–Fermi equation

$$\Delta\varphi(r) = -4\pi e\{\frac{[E_F - U_{\text{eff}}(r, p_F)]^3}{3\pi^2\hbar^3c^3} - Z\delta(\mathbf{r})\} \tag{17}$$

under specific boundary conditions.

This problem (for the boundary conditions $r\varphi(r) \to 0, r \to \infty$ and the simplified potential Eq. 10) was examined in Ref. 4 while analyzing the characteristics of a system of electrons in the field of a nucleus with charge Ze. It is obvious that such boundary conditions correspond to a free isolated atom not compressed by external forces. In the last case at $r \to \infty$, the potential energy $-e\varphi(r)$, the density of electrons $n_e(r)$, and the electron pressure P_e are equal to zero. The result is the presence of a shielded Coulomb potential in the volume of a "Thomas–Fermi atom" $\varphi(r) = \chi(r)Ze/r$, for which $\chi(r) = \exp(-r/R_{S(0)})$, $R_{S(0)} \approx 0.885(\hbar^2/m_ee^2)/Z^{1/3}$; $\chi(r) \to 1, r \to 0$; $\chi(r) \to 0, r \to \infty$. For such a "natural" isolated atom whose size and structure are determined only by the interaction between the nucleus and electrons, the self-compression is impossible.

Let's consider a basically different model: a compressed quasiatom whose size and structure are determined at the initial stage by external pressure.

Several works examined the characteristics of an extremely compressed degenerate electron gas surrounding the single nucleus. The screening distance $R_S = 5.8(3/4\pi n_e)^{1/3}$ for the nucleus in the infinite volume of a degenerate relativistic electron gas was calculated in Refs. 17, 18 by using the statistical dielectric function $\varepsilon_e(k, 0)$ (see Ref. 19). This result is not suitable for the following analysis of a compressed quasiatom with "ordinary" nucleus. It may be used only in the case where R_S is much less than the quasiatom radius: $R = Z^{1/3}(3/4\pi n_e)^{1/3}$ (i.e., $Z \gg 200$).

The model of compressed atom on the basis of the Thomas–Fermi equation was examined in Ref. 15. In particular, it follows from this model that, due to the increase in the concentrations of electrons and nuclei during compression, the role of the shielding effects becomes less substantial.

It is noted in Refs. 13–15 that, in the compressed environment with mass density $\rho \gg 10^4\,\text{g/cm}^3$ (it corresponds to the electron density $n_e \gg 3 \times 10^{27}\,\text{cm}^{-3}$) the state of a degenerate electron plasma corresponds to the ideal homogeneous gas with density n_e which is independent of coordinates. In this case, the Coulomb interaction may be taken into account as the interaction of this homogeneous gas with the field of a nucleus. The same result follows directly from Eq. 17 with R reduction.

These results simplify significantly the further calculations.

Consider a compressed quasiatom of radius R with the nucleus of charge Ze. The full Coulomb potential energy $U_{eQ} = U_{eQL} + U_{eQNL}$ of all Z degenerate electrons in the volume $V = 4\pi R^3/3$ consists from the linear (with respect to $e\varphi(r)$) part U_{eQL} and the nonlinear U_{eQN} (proportional to $(e\varphi(r)^2)$ one and may be deduced from Eq. 13 by summation of the contributions from all electrons according to their distribution within the Fermi sphere with the boundary (Fermi) momentum $p_F = (3\pi^2)^{1/3}\hbar n_e^{1/3}$.

The linear part of the potential energy is equal to

$$\begin{aligned} U_{eQL} &= \sum_{i=1}^{Z} U_{\rm eff}(r_i, p_i) \\ &= \int\limits_V \int\limits_0^{p_F} U_{\rm eff}(r,p)(d^2N/dpdV)dpdV = W_Q K_F, \qquad (18) \\ W_Q &= -\int\limits_V e\varphi(r) n_e dV, \qquad (19) \\ K_F &\equiv \langle\gamma\rangle = \frac{\int\limits_0^{p_F} (p^2c^2 + m_e^2c^4)^{1/2} p^2 dp}{\pi^2\hbar^3 n_e m_e c^2}. \qquad (20) \end{aligned}$$

Here, $d^2N/dpdV = p^2/\pi^2\hbar^3$ is the density of electron states in the r-space and p-space in the degenerate electron gas which is normalized by the full number of electrons:

$$\int\limits_V \int\limits_0^{p_F} (d^2N/dpdV)dpdV = Z. \qquad (21)$$

W_Q is the full Coulomb energy of electrons according to their homogeneous distribution in the volume of the compressed quasiatom but without regard for the influence of their movement in the Fermi condensate,
K_F is the factor of a dynamic increase in this energy related to the movement of electrons which is equal to the average value of the Lorentz factor $\langle\gamma\rangle$ for all electrons of the compressed quasiatom. For a relativistic degenerate gas of electrons, $\langle\gamma\rangle = (3/4)\gamma_F$.

The first part W_Q of the full energy U_{eQL} Eq. 18 is the sum of two components:

the attraction energy of a nucleus and electrons:

$$\begin{aligned} W_{Q(en)} &= \int\limits_0^R V_0(r) n_e 4\pi r^2 dr = -(3/2)(Z^2e^2/R) \\ &= -(3/2)(4\pi/3)^{1/3} Z^{5/3} e^2 n_e^{1/3} \qquad (22) \end{aligned}$$

and the repulsion energy of electrons:

$$\begin{aligned} W_{Q(ee)} &= \int\limits_0^R QdQ(r)/r = (3/5)Z^2e^2/R \\ &= (3/5)(4\pi/3)^{1/3} Z^{5/3} e^2 n_e^{1/3} \qquad (23) \end{aligned}$$

Here, $Q(r) = -Zer^3/R^3$ — the total charge of all homogeneously distributed degenerate electrons which are in the sphere with radius $r < R$ inside the compressed quasiatom.

The factor of dynamic increase of the full Coulomb energy is equal to

$$K_F = (m_e^3c^3/8\pi^2\hbar^3 n_e)\{(p_F/m_ec)[2(p_F/m_ec)^2+1] \times[(p_F/m_ec)^2+1]^{1/2} - Arsh(p_F/m_ec)\} \quad (24)$$

In the case of a nonrelativistic degenerate gas (at $p_F \ll m_ec$ and $n_e < 10^{30}\,\mathrm{cm}^{-3}$)

$$\begin{aligned} K_F &= 1+(3/10)(3\pi^2)^{2/3}(\hbar/m_ec)^2 n_e^{2/3} \\ &\approx 1+0.3(n_e/10^{30}\,\mathrm{cm}^{-3})^{2/3}. \end{aligned} \quad (25)$$

In the case of superrelativistic gas (at $p_F \gg m_ec$ and $n_e \gg 10^{30}\,\mathrm{cm}^{-3}$)

$$\begin{aligned} K_F &= (3/4)(3\pi^2)^{1/3}(\hbar/m_ec) n_e^{1/3} \\ &\approx 0.77(n_e/10^{30}\,\mathrm{cm}^{-3})^{1/3}. \end{aligned} \quad (26)$$

Accordingly, the nonlinear part of the full Coulomb potential energy reads

$$\begin{aligned} U_{eQNL} &= \int_V [e\varphi(r)]^2 n_e dV/2m_ec^2 \\ &= \int_0^R (-Ze^2/r + Ze^2r^2/R^3)^2 n_e 4\pi r^2 dr/2m_ec^2 \\ &= [8\pi Z^{7/3}e^4(3/4\pi)^{1/3}/7m_ec^2] n_e^{2/3}. \end{aligned} \quad (27)$$

A change of the full Coulomb energy of the degenerate gas of electrons Eq. 22 and Eq. 23 is accompanied by a simultaneous change of the exchange energy of the degenerate electron gas (see Ref. 20)

$$\begin{aligned} U_{e,exch} &= -(e^2/n_e)\int_0^{p_F} d^3p_1/(2\pi\hbar)^3 \times \int_0^{p_F} d^3p_2/(2\pi\hbar)^3 \frac{4\pi\hbar^2}{|\mathbf{p}_1-\mathbf{p}_2|^2} \\ &= -[3(3\pi^2)^{1/3}/4\pi] Ze^2 n_e^{1/3} \end{aligned} \quad (28)$$

and an increase in its kinetic (Fermi) energy

$$\begin{aligned} U_{eF} &= \int_V \int_0^{p_F} T_1(p)(dN^2/dpdV)dpdV \\ &= (V/\pi^2\hbar^3)\int_0^{p_F} [(p^2+m_e^2c^2)^{1/2} - m_ec^2]p^2dp \\ &= Zm_ec^2(K_F-1). \end{aligned} \quad (29)$$

The total energy of the degenerate electron gas is

$$\begin{aligned} U_{e\Sigma} &= U_{eQL} + U_{eQNL} + U_{e,exch} + U_{eF} \\ &= [W_{0(en)} + W_{0(ee)} + Zm_ec^2]K_F + U_{e,exch} \\ &\quad - Zm_ec^2 + U_{eQNL} \\ &= \{Zm_ec^2 - (9/10)(4\pi/3)^{1/3}Z^{5/3}e^2n_e^{1/3}\}K_F \\ &\quad - [3(3\pi^2)^{1/3}/4\pi]Ze^2n_e^{1/3} - Zm_ec^2 \\ &\quad + [8\pi Z^{7/3}e^4(3/4\pi)^{1/3}/7m_ec^2]n_e^{2/3}. \end{aligned} \tag{30}$$

It is easy to see from these results that, for the relativistic degenerate gas, the nonlinear component of energy U_{eQNL} for a nucleus with $Z \ll 2800$ is much less than the linear component U_{eQL} and can be omitted for such nuclei.

2.3 Peculiarities of the Evolution and Collapse of a Neutral Wigner–Seitz Cell at High Densities of the Compressed Relativistic Degenerate Gas of Electrons

From Eq. 30 the effective total pressure of the electron gas on the quasiatom surface may be found as

$$P_e(Z, n_e) = -dU_{e\Sigma}/dV = (n_e^2/Z)dU_{e\Sigma}/dn_e. \tag{31}$$

The dependence of the energy $U_{e\Sigma}$ Eq. 30 of the degenerate electron gas on its density n_e for nuclei with $Z = 29$, 50, 70, 82, 92 per electron is shown in Fig. 3 . . . Fig. 6 for three ranges of the degenerate electron gas density n_e ($10^{24} \ldots 3 \times 10^{28}\,\mathrm{cm}^{-3}; 10^{29} \ldots 3 \times 10^{33}\,\mathrm{cm}^{-3}; 10^{32} \ldots 10^{35}\,\mathrm{cm}^{-3}$).

It follows from Fig. 3 that, at a rather low density of the nonrelativistic degenerate gas (in the range $n_e \approx (0.8 \ldots 3) \times 10^{27}\,\mathrm{cm}^{-3}$), a local minimum of energy corresponds to each type of nuclei. These minima correspond to zero pressure $P_e(Z, n_e) = 0$ on the surface of a compressed quasiatom. The condition $P_e(Z, n_e) = 0$ determines steady states of the compressed quasiatom which may exist without any external influence. They are analogs (in the area of high electron densities) of a metal bond in crystals which is realized in a degenerate nonrelativistic electron gas of conductivity electrons at their low density $n_e \approx 10^{23}\,\mathrm{cm}^{-3}$ in metals. It should be noted that, in the case of a neutral degenerate electron-ion plasma containing identical nuclei (Fig. 2), the same pressure will be applied to the external surface of this plasma.

Under a further compressing of the electron-nuclear plasma, the density n_e and energy of the electron gas increase, and the steady states are disrupted, which corresponds to the transition to an unstable plasma. For the confinement of such a plasma, it is necessary to apply an additional (negative) external pressure equal to the positive pressure on the surface of a compressed quasiatom $P_e(Z, n_e) > 0$. At a further increase in the density n_e (up to $n_{e(cr)} \approx 10^{32} \ldots 10^{33}\,\mathrm{cm}^{-3}$, which corresponds to a relativistic gas), the full energy of the compressed quasiatom reaches the maximum which corresponds

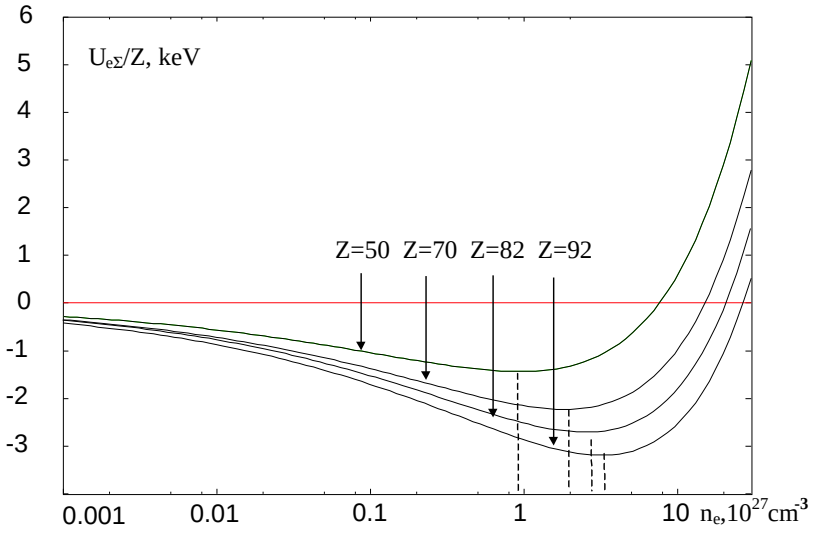


Figure 3: Degenerate electron gas energy in the low-density area in the volume of a compressed stable quasiatom.

to zero pressure $P_e(Z, n_{e(cr)}) = 0$ on the surface of a Wigner–Seitz cell and to the state of unstable balance between the Fermi repulsion forces of electrons and the Coulomb attraction of electrons to a nucleus. The structure of the energy maximum is presented in Fig. 4 (for the average energy of each electron, $U_{e\Sigma}/Z$) and Fig. 5 (for the full energy of the degenerate gas, $U_{e\Sigma}$).

A further increase in the electron gas density reduces the full energy of the electron gas, which corresponds to the negative pressure $P_e(Z, n_e) < 0$ on the surface of a Wigner–Seitz cell. Pressure of this kind renders the compressed quasiatom at $n_e > n_{e(cr)}$ to be unstable relative to the spontaneous self-compression of the plasma and to the subsequent unlimited increase in the electron density n_e. In this case, the compression of the relativistic degenerate gas occurs without loss of its ideality. This self-reinforcing process of irreversible self-compression of the degenerate electron gas is accompanied by "the downfall of electron shells of the compressed quasiatom to the nucleus" and results in the full collapse of the electron-nuclear plasma in the volume of a particular Wigner–Seitz cell. The dependence of the average energy of the degenerate relativistic electron plasma in the stage of formation of a self-reinforcing collapse is shown in Fig. 6. It can be seen from this figure that the average binding energy of each electron may reach tens of MeV.

The threshold value $n_{e(cr)} \equiv n_{e(cr)}(Z)$, at which the pressure inversion and the formation of the collapse state take place, depends on the charge of a nucleus and monotonously increases with reduction in Z, starting from $n_{e(cr)} \approx 2 \times 10^{32}\,\mathrm{cm}^{-3}$ for the heaviest stable nucleus (a nucleus of uranium with $Z = 92$). For copper with $Z = 29$, $n_{e(cr)} \approx 2 \times 10^{33}\,\mathrm{cm}^{-3}$.

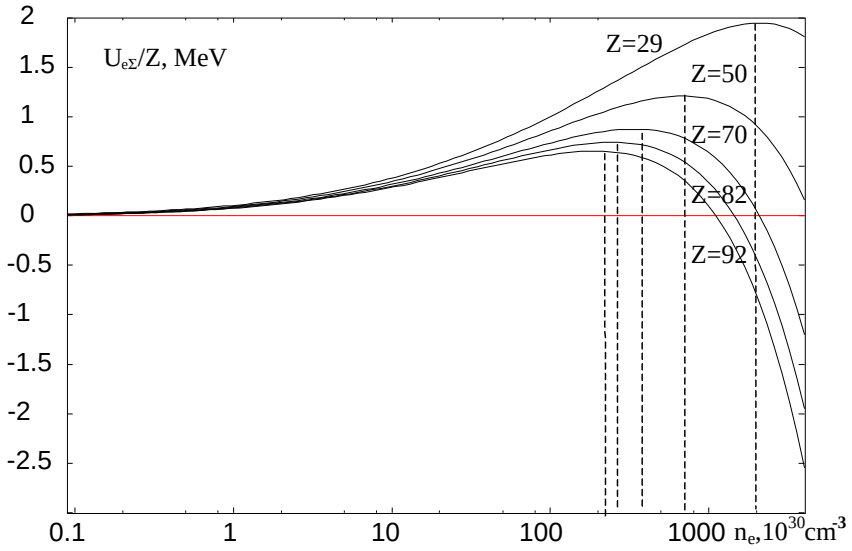


Figure 4: Degenerate electron gas energy in the volume of a compressed stable quasiatom in the area of unstable balance.

It seems at first sight that the necessary values $n_{e(cr)} \geq 2 \times 10^{32}\,\mathrm{cm}^{-3}$ for the critical density of a degenerate electron gas are so great that they hardly can be reached. In fact, the situation is different.

The matter is that the defining parameter is not $n_{e(cr)}$, but the additional energy which is necessary for a preliminary compression of the electron-nuclear plasma up to this density.

It follows from the calculations carried out that a rather small energy is necessary for reaching the collapse threshold of a compressed quasiatom (the threshold density of electrons $n_{e(cr)}$). The average energy $U_{e\Sigma}/Z \approx 0.65\,\mathrm{MeV/electron}$ is necessary for a nucleus with $Z = 92$ for reaching the threshold density $n_{e(\mathrm{cr})} \approx 2 \times 10^{32}\,\mathrm{cm}^{-3}$. This value is much less than the kinetic (Fermi) energy

$$U_{eF}/Z = (3/4)(3\pi^2)^{1/3}\hbar c n_{e(\mathrm{cr})}^{1/3} \approx 3\,\mathrm{MeV/electron} \tag{32}$$

required for the degenerate electron gas to reach $n_{e(\mathrm{cr})}$ without regard for the additional influence of the Coulomb attraction of nuclei. The difference of these values $U_{eF}/Z - U_{e\Sigma}/Z \approx 2.4\,\mathrm{MeV/electron}$ corresponds to a spontaneous increase of the binding energy of electrons with a nucleus in the volume of a compressed quasiatom. The physical explanation of such a favorable effect is related to the fact that, under an additional compression of the degenerate electron gas alongside with increase in its kinetic energy (which increases the full energy), the potential and full energies are reduced because, at such a compression, the electrons are located in the area of the stronger Coulomb field around a nucleus.

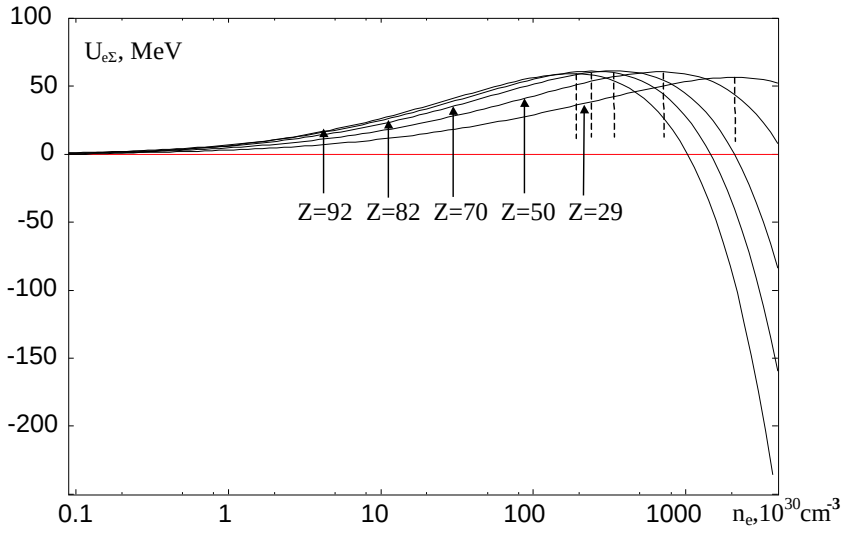


Figure 5: Full energy of the degenerate superrelativistic electron gas in the volume of a compressed stable quasiatom in the area of unstable balance.

Thus, about 80 % of the activity necessary for reaching the collapse threshold is carried out by the electron-nuclear system on its own through the Coulomb interaction. Without this effect, at the expense of the energy of an external force $U_{e\Sigma}/Z \approx 0.65\,\mathrm{MeV/electron}$, it is possible to reach the degenerate electron gas density

$$n_{e(\mathrm{eff})} \approx [(64/81\pi^2)/(\hbar c)^3]U_{e\Sigma}/Z \approx 10^{30}\,\mathrm{cm}^{-3}, \tag{33}$$

which is by 200 times less than that gained with the help of electron-nuclear Coulomb interaction.

Accordingly, the average energy $U_{e\Sigma}/Z \approx 0.75\,\mathrm{MeV/electron}$ is necessary for a nucleus of lead ($Z = 82$) to reach the critical density $n_{e(\mathrm{cr})} \approx 3 \times 10^{32}\,\mathrm{cm}^{-3}$. Without taking into account the Coulomb interaction, such an average energy allows one to reach the electron gas density

$$n_{e(\mathrm{eff})} \approx [(64/81\pi^2)/(\hbar c)^3]U_{e\Sigma}/Z \approx 1.2 \times 10^{30}\,\mathrm{cm}^{-3}. \tag{34}$$

The dependence of the critical density on the charge of a nucleus results in the certain sequential stages of the transition of a nucleus to the state of collapse. Under a gradual increase in the external pressure, the first to pass to the state of collapse will be the electrons from the volumes of those Wigner–Seitz cells whose centers possess nuclei with the maximal charge. If the degenerate electron plasma contains different nuclei, then only those nuclei will pass to the state of electron-nuclear collapse which satisfy the condition of pressure inversion at the appropriate critical density $n_{e(\mathrm{cr})}(Z)$ in the volume. Thus, the relativistic degenerate electron-nuclear plasma may contain heavy nuclei in the state of

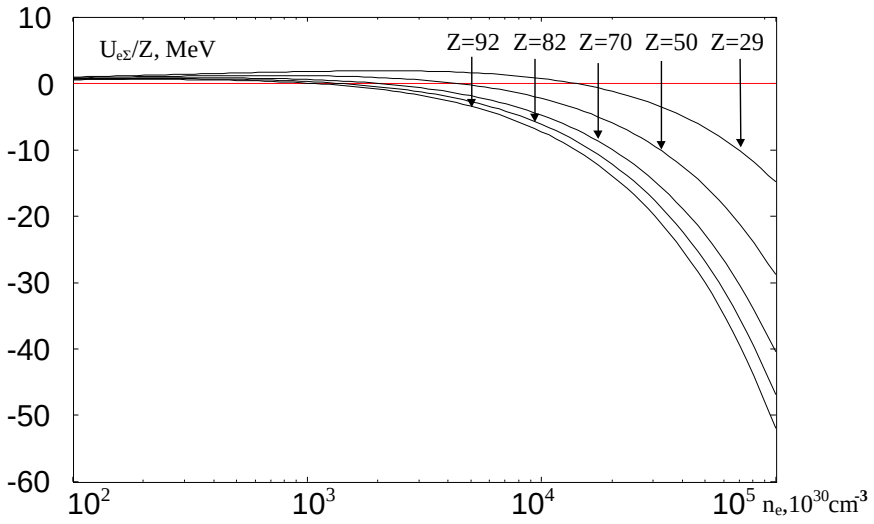


Figure 6: Full energy of the degenerate superrelativistic electron gas in the volume of a compressed stable quasiatom in the stage of self-compression.

electron-nuclear collapse as well as lighter nuclei which belong to compressed, but not collapsed quasiatoms.

3 SYNTHESIS OF SUPERHEAVY NUCLEI AND PECULIARITIES OF THE FORMATION OF A NUCLEAR CLUSTER

The process of formation of the electron-nuclear collapse begins after reaching the critical electron density $n_{e(\mathrm{cr})}$ and leads to an avalanche-type increase in the density of electrons $n_e \gg n_{e(\mathrm{cr})}$ in the degenerate relativistic gas up to the value comparable to the density of protons n_p in a nucleus. In such a process, there is a substantial increase in the binding energy of the electrons participating in the electron-nuclear collapse, which changes both the nature of intranuclear processes and the further evolution of the collapse.

First of all, it should be noted that the evolution of the collapse should be examined with regard for the full energy of a compressed quasiatom, including both the energy of a nucleus E_n Eq. 1 and the full energy of electrons $U_{e\Sigma}$ Eq. 30. In addition, the expression for the nucleus energy should be changed in view of the partial screening of the proton charge inside a nucleus by the degenerate gas of relativistic electrons, which is equivalent to a reduction of the charge of each proton ($e \to [1 - n_e/n_p]e$) or the number of protons ($Z \to [1 - n_e/n_p]Z$).

Fig. 7 demonstrates the dependence of the full energy per one nucleon of a compressed quasiatom (Wigner–Seitz cells are in the phase of formation of the

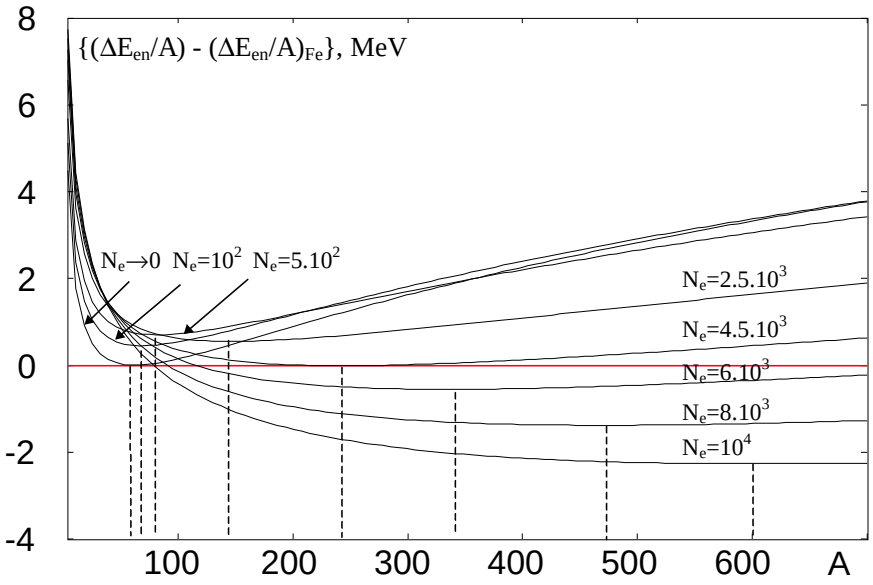


Figure 7: Energy of a compressed quasiatom in the state of partial collapse of the electron-nuclear degenerate plasma for different concentrations of the degenerate superrelativistic electron gas $N_e = (n_e/10^{30}\ \mathrm{cm}^{-3})$.

electron-nuclear collapse)

$$\begin{aligned} \Delta E_{en}/A &= \varepsilon_2/A^{1/3} + \varepsilon_3(1-n_e/n_p)^2 Z_{\mathrm{opt}}^2/A^{4/3} \\ &\quad + \varepsilon_4[1/2 - (1-n_e/n_p)Z_{\mathrm{opt}}/A]^2 \\ &\quad + \varepsilon_5/A^{7/4} + U_{e\Sigma}(Z_{\mathrm{opt}})/A \end{aligned} \tag{35}$$

on both A and the degenerate relativistic electron gas density in the range $1 < A < 700$.

In calculations, we used the classical line of stability $Z_{\mathrm{opt}}(A)$ for the charge of a nucleus by taking the screening into account:

$$\begin{aligned} Z_{\mathrm{opt}} &= \frac{A}{2[1 + (\varepsilon_3/\varepsilon_4)(1-n_e/n_p)^2 A^{2/3}]} \\ &\approx \frac{A}{2 + 0.0155(1-n_e/n_p)^2 A^{2/3}}. \end{aligned} \tag{36}$$

For a convenience of the analysis, the value $\Delta E_{en}/A$ was normalized to the energy of the most stable nuclei such as iron, the constant total energy of the rest of all nucleons and electrons, and the binding energy of all nucleons $\varepsilon_1 A$.

The results of calculations yield that, by increasing the density of electrons in the volume of a nucleus, the energy of the electron-nuclear system continuously

reduces (this is caused by the corresponding increase of its binding energy), and the minimum of this energy shifts to the area of big masses A_m.

At $n_e < 10^{30}\,\mathrm{cm}^{-3}$ ($N_e \equiv n_e/10^{30}\,\mathrm{cm}^{-3} < 1$), the minimum of the electron-nuclear system energy corresponds to nuclei with mass number $A_{\mathrm{opt}} \approx 60$. With increase in the electron density, the minimum of the full energy A_{opt} increases as well (see Table 1).

Table 1: Atomic mass A_{opt} corresponding to the minimum of the full energy of the electron-nuclear system with electron density n_e

n_e, cm^{-3}	A_{opt}
$< 10^{30}$	≈ 60
$< 10^{32}$	≈ 70
$< 5 \times 10^{32}$	≈ 80
$< 2.5 \times 10^{33}$	≈ 150
$< 4.5 \times 10^{33}$	≈ 240
$< 6 \times 10^{33}$	≈ 340
$< 8 \times 10^{33}$	≈ 470
$< 10^{34}$	≈ 600
$< 10^{35}$	≈ 2000

It is obvious that, at reaching the density $n_e > 10^{35}\,\mathrm{cm}^{-3}$, the minimum of the electron-nuclear system energy shifts to $A_{\mathrm{opt}} > 3000$, where the threshold state for the existence of a nucleus with pion condensate becomes possible.

The position shift of A_{opt} for a minimum of the full energy E_{en}/A is accompanied by a change of the energy of this minimum from the initial value $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} = 0$ at $n_e \ll 10^{30}\,\mathrm{cm}^{-3}$ (this corresponds to nuclei of the iron group) at first to the side of increasing the minimum up to $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx 0.65\,\mathrm{MeV/nucleon}$ at $n_e = 2.5 \times 10^{32}\,\mathrm{cm}^{-3}$. Then its irreversible reduction occurs: $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx 0$ at $n_e = 4.5 \times 10^{33}\,\mathrm{cm}^{-3}$, $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx -1.4\,\mathrm{MeV/nucleon}$ at $n_e = 8{\times}10^{33}\,\mathrm{cm}^{-3}$, $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx -2.2\,\mathrm{MeV/nucleon}$ at $n_e = 10^{34}\,\mathrm{cm}^{-3}$.

The derived values of A_{opt} and $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}}$ characterize the direct influence of degenerate electrons (owing to the self-energy of the electron condensate).

Fig. 8 shows the dependence of the same parameter $\Delta E_{en}/A$ in a greater range of A and n_e. It is seen that, with increase in the electron density up to $n_e \approx 10^{35}\,\mathrm{cm}^{-3}$, this minimum reduces down to $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}} \approx -30\,\mathrm{MeV/nucleon}$.

Such a shift of the position and depth of the energy minimum of the electron-nuclear system opens a way to the formation of extremely heavy nuclei through the repeated absorption of "ordinary" nuclei by the nucleus in a state of collapse.

It is necessary to note that even greater shifts of a minimum of the full energy A_{opt} and $\Delta E_{en}(A_{\mathrm{opt}})/A_{\mathrm{opt}}$ are possible. The reason for this is that the classical line of stability $Z_{\mathrm{opt}}(A)$ for the charge of a nucleus Eq. 36 corresponds to the optimization condition of nuclear parameters on the basis of the formula

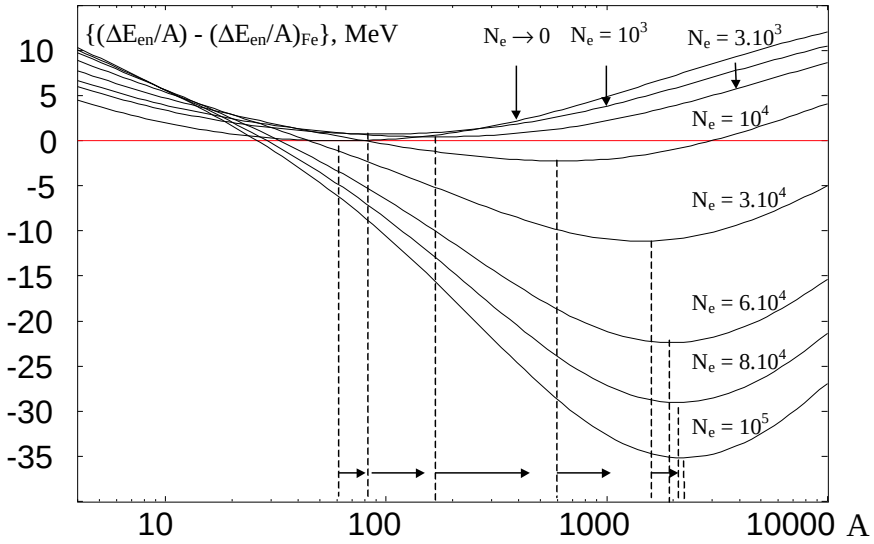


Figure 8: Energy of a compressed quasiatom in the state of partial collapse of the electron-nuclear degenerate plasma for different concentrations of the degenerate superrelativistic electron gas $N_e = (n_e/10^{30}\ \mathrm{cm}^{-3})$. Arrows in the lower part of the figure show the direction of a shift of the full energy minimum (or binding energy maximum) of a compressed quasiatom.

$d(E_n/A)/dZ = 0$. If the optimization involves both nuclear and electron parameters with regard for the condition $d(E_e n/A)/dZ = 0$, then we can find the corrected value $Z_{\rm opt}(A, n_e)$ which corresponds to a deeper minimum of the electron-nuclear system energy.

For example, at the density of electrons $n_e \ll 10^{42}/A^3Z^2\,{\rm cm}^{-3}$, the optimum charge of a nucleus (and the number of electrons in a compressed quasiatom) is determined by

$$Z_{\rm opt}(A, n_e) = \frac{A[1 + ABZ^{2/3}n_e^{1/3}]}{2[1 + (\varepsilon_3/\varepsilon_4)(1 - n_e/n_p)^2A^{2/3}]}\ , \tag{37}$$

$$B = (9/10)(4\pi/3)^{1/3}e^2/\varepsilon_4 \approx 10^{-14}\ {\rm cm}. \tag{38}$$

The synthesis of superheavy nuclei is facilitated also by the fact that the electric field of a nucleus on the boundary of a compressed quasiatom (on the boundary of a Wigner–Seitz cell) is completely neutralized by the condensed electrons. The reduction of the compressed quasiatom radius in its collapse greatly reduces the influence of the Coulomb barrier on the synthesis of nuclei.

By a moderately strong compression of the degenerate relativistic gas of electrons (up to the density $n_{e({\rm cr})} \approx 10^{32}\,{\rm cm}^{-3} < n_e \ll n_p \approx 10^{38}\,{\rm cm}^{-3}$ which is much less than the density of protons n_p), the width of the Coulomb barrier is strongly decreased: from the initial value equal to the screening radius $R_{S(0)} \approx 0.885(\hbar^2/m_e e^2)/Z^{1/3}$ in atoms and ions with a small degree of ionization or in the uncompressed degenerate electron plasma up to $R_S \approx R = (3Z/4\pi n_e)^{1/3}$ determined by the radius R of a Wigner–Seitz cell. In particular, for a nucleus with $Z = 92$, the initial screening radius $R_{S(0)} \approx 0.1\,{\rm \AA}$. The Coulomb barrier width decreases at the density $n_{e({\rm cr})} \approx 10^{32}\,{\rm cm}^{-3}$ by $R_{S(0)}/R_S \approx 16.6$ times, at the density $n_{e({\rm cr})} \approx 10^{33}\,{\rm cm}^{-3}$ by $R_{S(0)}/R_S \approx 36$ times, at the density $n_{e({\rm cr})} \approx 10^{35}\,{\rm cm}^{-3}$ by $R_{S(0)}/R_S \approx 166$ times, and at the density $n_{e({\rm cr})} \approx 10^{37}\,{\rm cm}^{-3}$ by $R_{S(0)}/R_S \approx 775$ times. As the probability of a tunnelling transition of a nucleus through the Coulomb barrier in the compressed electron-nuclear plasma depends basically on the barrier width, then such a drastic reduction of the barrier width increases greatly the probability of the synthesis.

There are other estimations proving the efficiency of the synthesis.

It is possible to characterize the nuclei of a cold compressed electron-nuclear plasma by the effective energy of relative movement $T_{\rm eff}$ which should be possessed by the similar, but free nuclei in order that they can approach one another to the same distance R_S. For a pure Coulomb nonscreened interaction of two nuclei with charges Z_1 and Z_2 (it corresponds to the examined case of relativistic degenerate electron gas), this energy is determined by $T_{\rm eff} = Z_1Z_2e^2/R_S$. For example, in order that a free nucleus of lead and a proton approach one another to the the distance $R_{S(0)} \approx 1.2 \times 10^{-12}\,{\rm cm}$ (it corresponds to the density $n_e \approx 10^{36}\,{\rm cm}^{-3}$), the relative energy $T_{\rm eff} \approx 15\,{\rm MeV}$ (or $15\,{\rm MeV/nucleon}$) is required. Accordingly, the interaction of two lead nuclei in a degenerate electron gas with the same density requires the energy $T_{\rm eff} \approx 1400\,{\rm MeV}$ (or the same $15\,{\rm MeV/nucleon}$). With increase in the density of electrons, $T_{\rm eff}$ grows. Such an energy renders the probability of the tunnelling effect to be high.

Such a synthesis has one additional effect.

In the process of such a fusion, the excessive energy is released via different channels such as emission of gamma quanta, neutrons, nuclear fragments, etc. One of the channels is connected with the creation of different "ordinary" nuclei and the emission of these nuclei from the volume of a growing superheavy nucleus. For example, after the absorption of several nuclei with $A_T \approx 50 \dots 200$ in a short time, high binding energy can lead to the emission of several light nuclei with $A_L < A_T$ or one heavy nucleus with $A_H \approx 300 \dots 500 > A_T$.

Such a process of nuclei emission competes with other channels of cooling of the nuclear substance. In this case, even-even nuclei (He^4, C^{12}, O^{16}, ..., Pb^{208}) which already exist in the volume of a superheavy nucleus are more likely to emerge and be emitted. In fact, every superheavy nucleus in the phase of formation of an electron-nuclear collapse may be a "specific microreactor" for the transmutation of "ordinary" nuclei to different configurations of nucleons. In this microreactor, the process of transmutation comes to the end after the utilization of all neighboring nuclei or after the evolution of a superheavy nucleus to the final stable state with $A_{\max}$.

In conclusion, it should be noted that the problem of self-compression of the system of degenerate relativistic electrons being in a compressed quasiatom (i.e., the achievement of the maximal ratio n_e/n_p and the weakening of interactions between protons inside a nucleus) may be solved only by performong the full analysis of the whole variety of electron-nuclear transformations (including the processes of protonization and neutronization and the formation of electron-positron pairs) for a specific nucleus. At the same time, it is clear that the boundary density of the compressed electron gas $n_{e(max)}$ will always be less than the density of protons in a nucleus n_p because the only reason for the compression of the relativistic degenerate gas of electrons is its interaction with the effective charge of a nucleus which decreases in the process of compression $Z^* = Z(n_p - n_e)/n_p$.

4 SUMMARY

We have proposed a theoretical model of a threshold-type self-similar electron-nuclear collapse gained at reaching the energy density in the electron plasma of about 0.65 MeV/electron, which corresponds to the high critical density of the degenerate electron gas $n_{e(\mathrm{cr})}$. This density is very high as compared with the value of $n_e \approx 10^{24}\,\mathrm{cm}^{-3}$ typical, for example, of the natural state of the conductivity zone in metals. At the same time, the value $n_{e(\mathrm{cr})}$ appears to be less by many orders of magnitude than the final density $n_e \approx 10^{36} \dots 10^{37}\,\mathrm{cm}^{-3}$ which is automatically reached after a preliminary compression in the self-organizing process of formation of the collapse state and is comparable to the intranuclear density of nucleons n_n. In the collapse state, the direction and probability of nuclear reactions are changed significantly, the influence of the Coulomb barrier is greatly reduced, and the spontaneous synthesis of superheavy stable nuclei becomes probable. This approach differs basically from the "force" method of