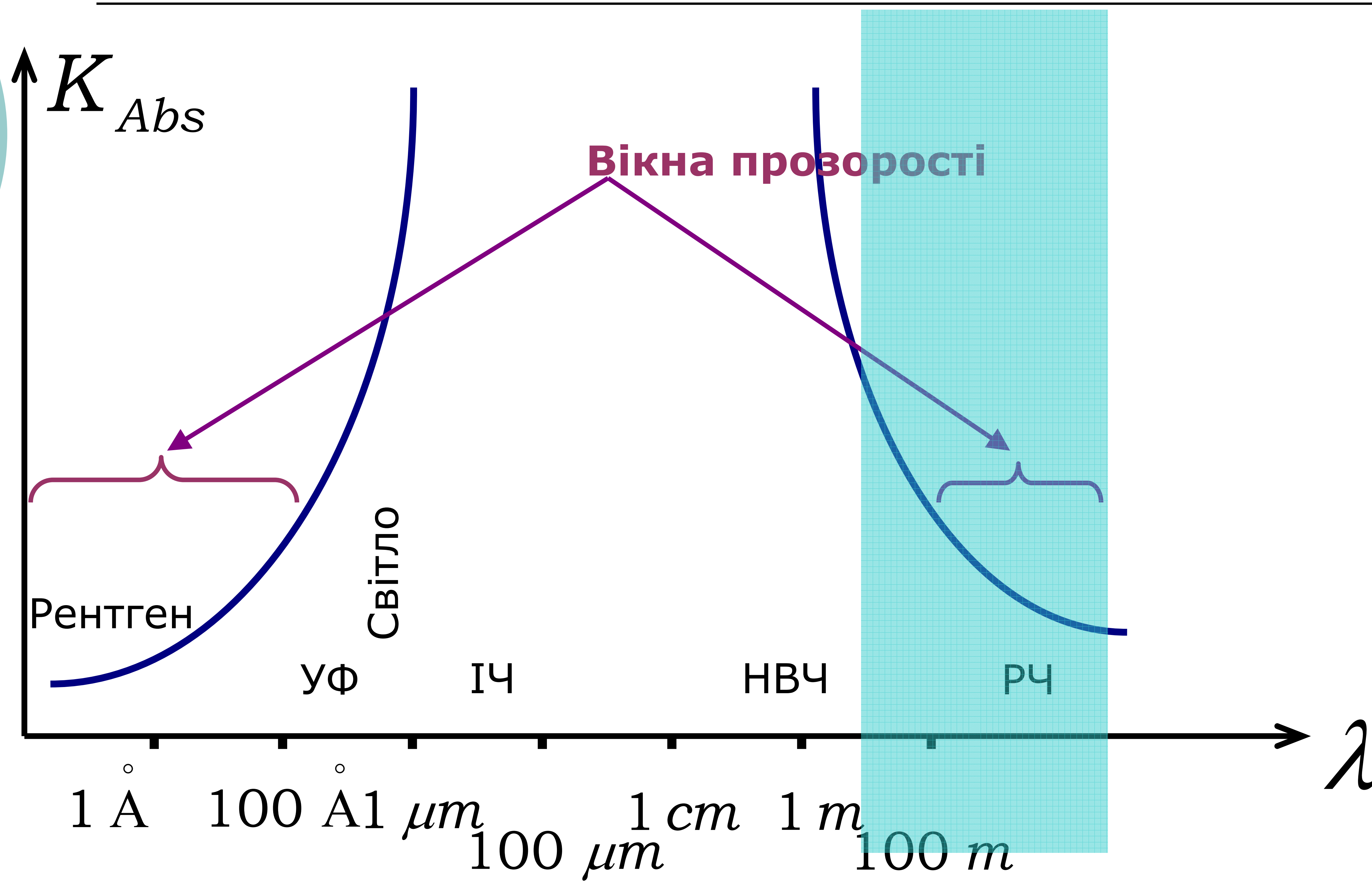
Медична радіофізика

Лекція 10.

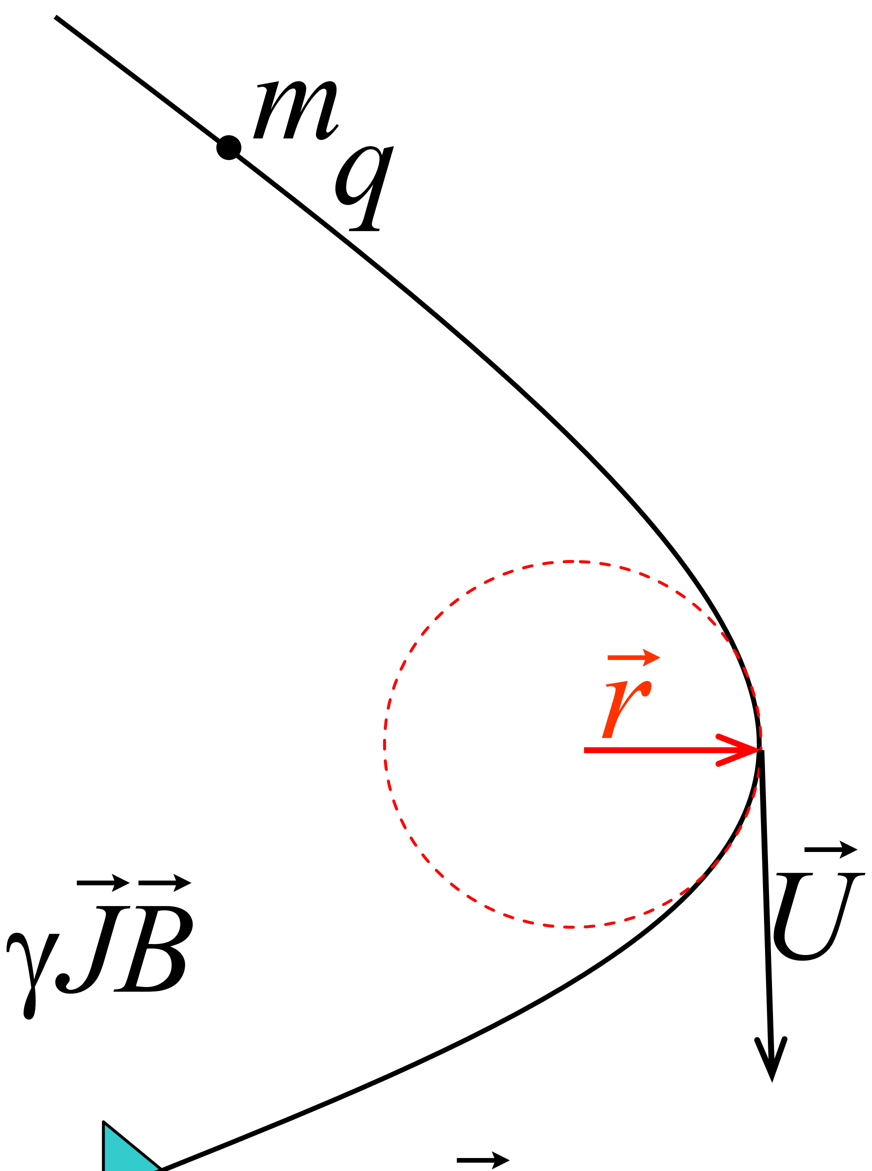
МР-томографія.

РЧ.



K_{Abs}
Вікна прозорості
Рентген
Світло
УФ
ІЧ
НВЧ
РЧ
λ
1 Å
100 Å
1 μm
100 μm
1 cm
1 m
100 m

Механічний і магнітний моменти.



$\odot\ \vec{B}$

$$\varepsilon = -\vec{\mu}\vec{B} = -\gamma\vec{J}\vec{B}$$

$$\vec{J} = \left[\vec{r} \times m\vec{U}\right]$$

$$\vec{\mu} = \frac{1}{2c}\left[\vec{r} \times q\vec{U}\right]$$

$$\vec{\mu} = \gamma\vec{J}$$

$$\gamma = \frac{q}{2mc}$$

$$\vec{M} = \left[\vec{\mu} \times \vec{B}\right]$$

$$\frac{d\vec{J}}{dt} = \vec{M} = \left[\vec{\mu} \times \vec{B}\right]$$

Взаємодія.

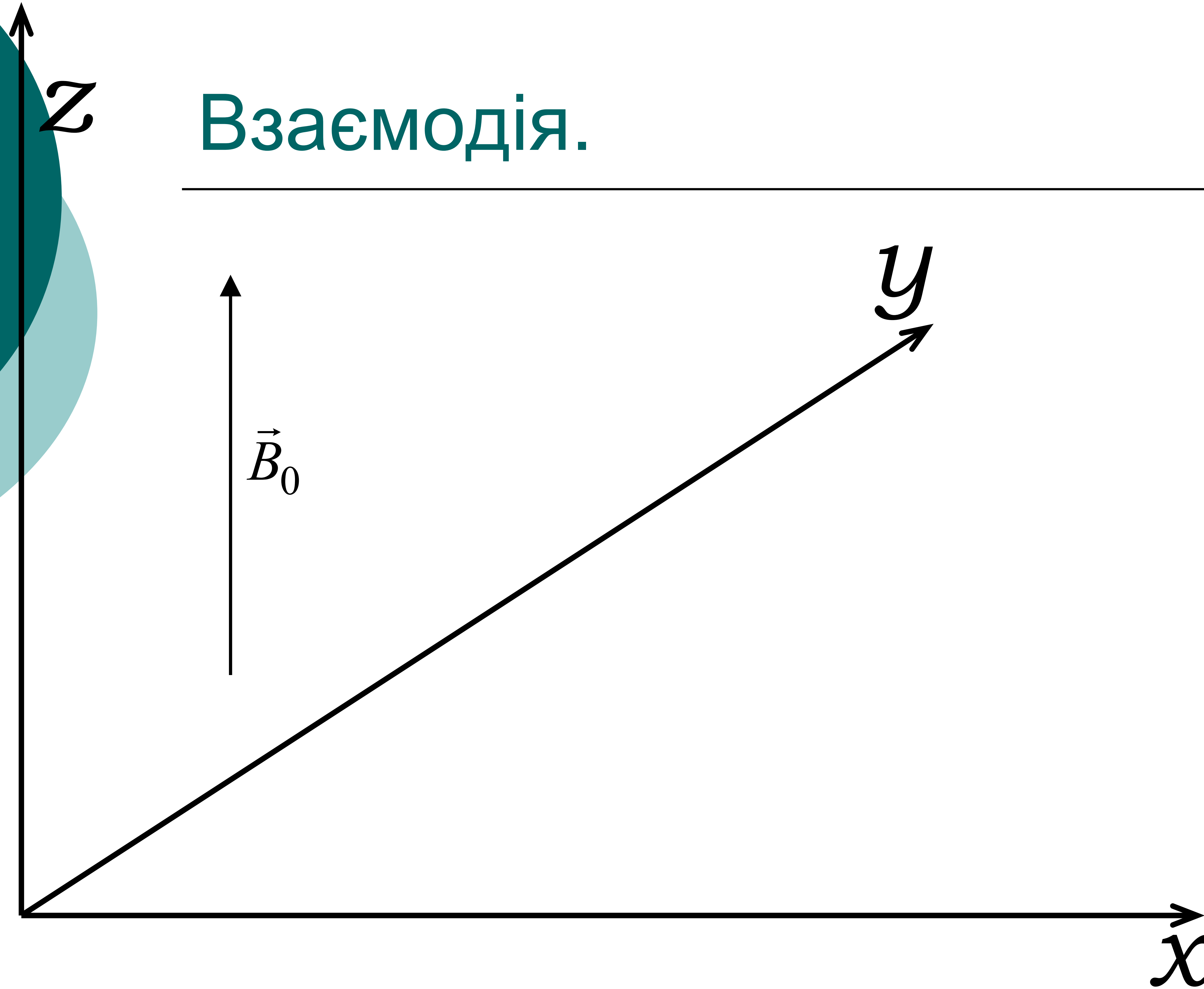
$$\frac{d\vec{\mu}}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]$$

$$\frac{d\mu_x}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_x = \gamma\left(\mu_y B_z - \mu_z B_y\right)$$

$$\frac{d\mu_y}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_y = \gamma\left(\mu_z B_x - \mu_x B_z\right)$$

$$\frac{d\mu_z}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_z = \gamma\left(\mu_x B_y - \mu_y B_x\right)$$

Взаємодія.



Взаємодія.

$$\frac{d\vec{\mu}}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right] \quad \vec{B} = \left\{B_x, B_y, B_z\right\} = \left\{0, 0, B_0\right\}$$

$$\frac{d\mu_x}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_x = \gamma\mu_y B_0$$

$$\frac{d\mu_y}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_y = -\gamma\mu_x B_0$$

$$\frac{d\mu_z}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_z = 0$$

Взаємодія.

$$\frac{d\vec{\mu}}{dt} = \gamma\left[\vec{\mu} \times \vec{B}\right]$$

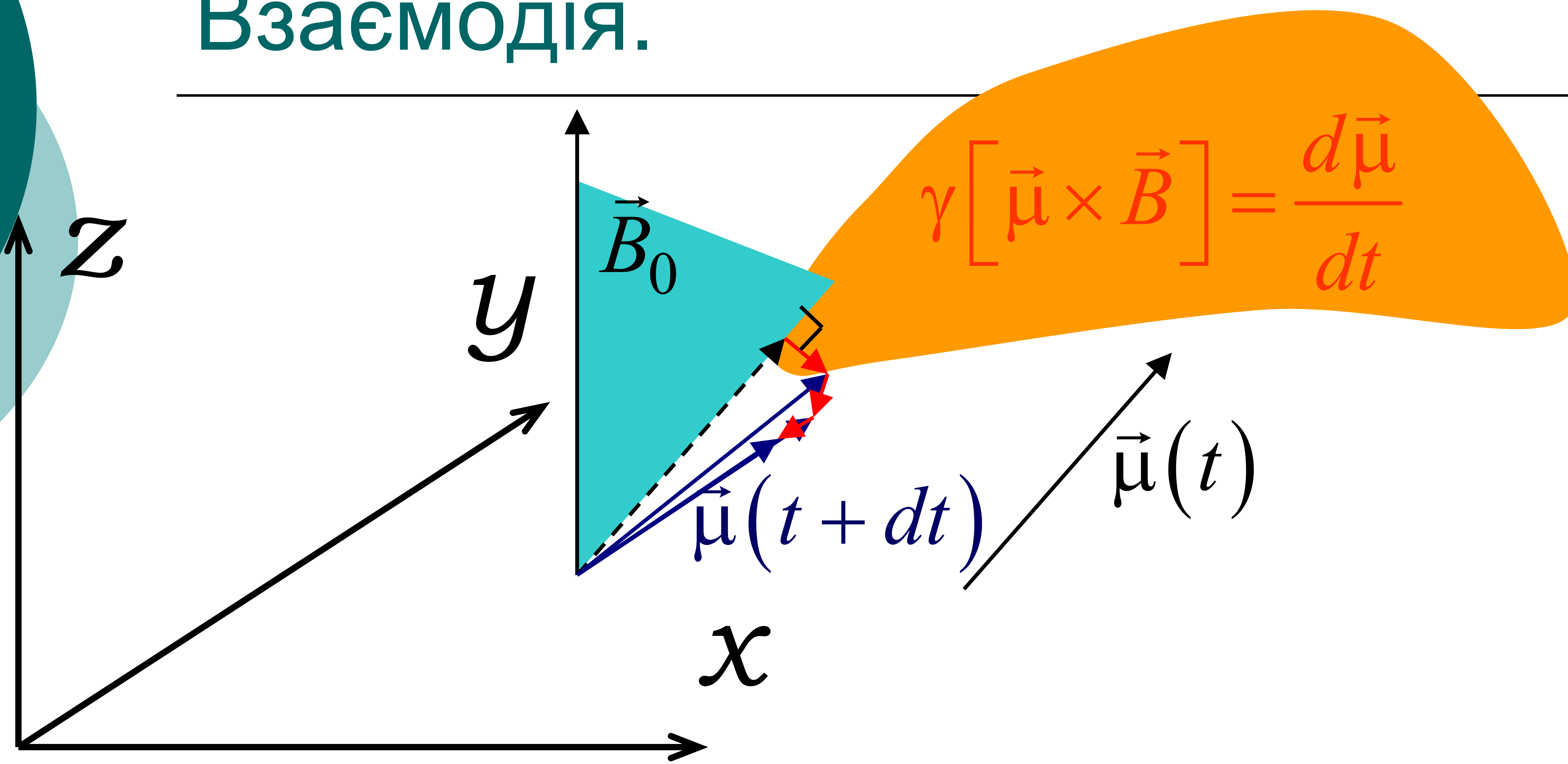
$$\mu_x(t) = \mu_x^{(0)} \cos\omega_0 t + \mu_y^{(0)} \sin\omega_0 t$$

$$\mu_y(t) = \mu_y^{(0)} \cos\omega_0 t - \mu_x^{(0)} \sin\omega_0 t$$

$$\mu_z(t) = \mu_z^{(0)} = \mu_\parallel \neq f(t)$$

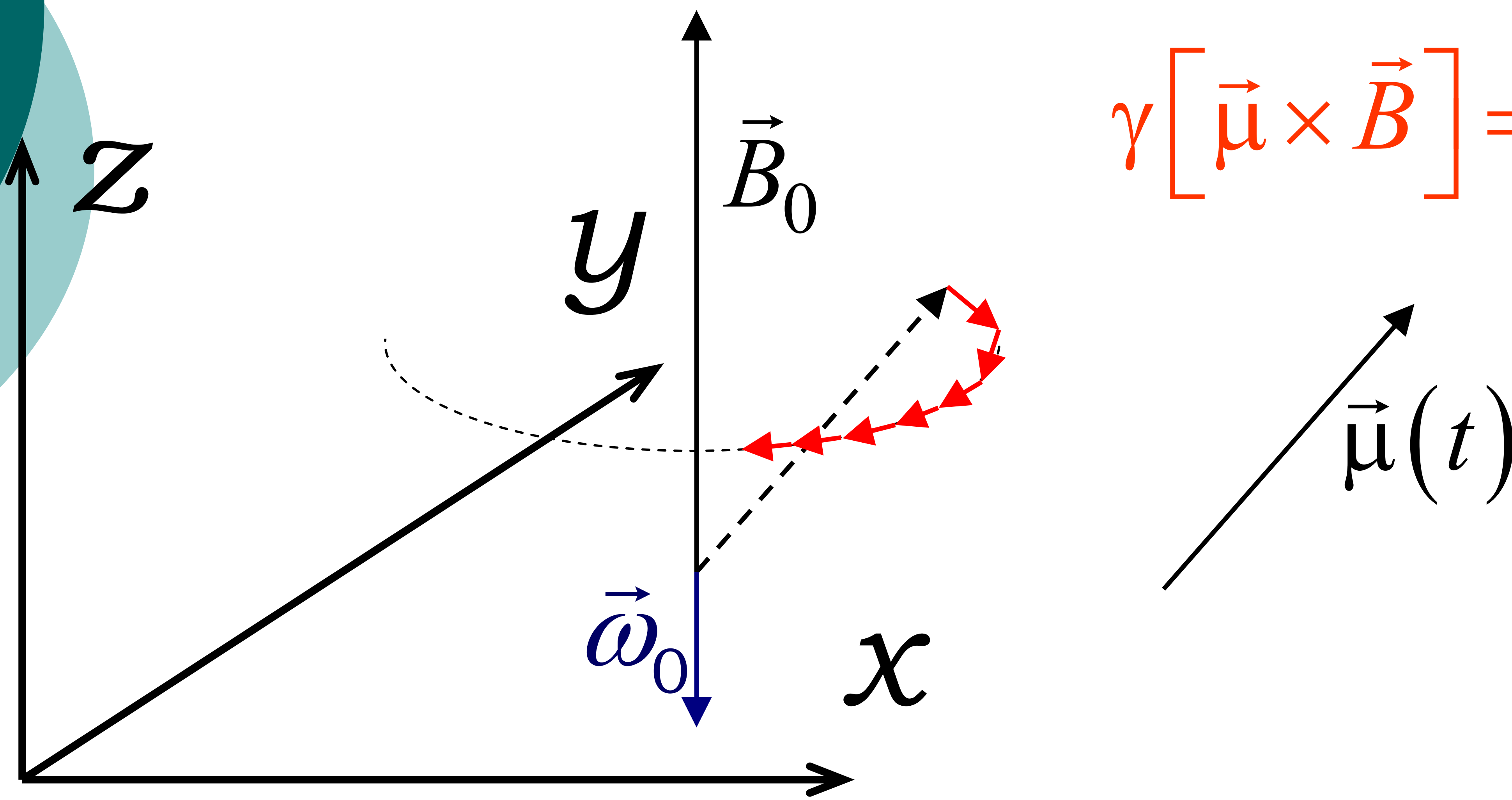
$$\omega_0 = -\gamma B_0$$

Взаємодія.



$$\frac{d\vec{\mu}}{dt}=\gamma\left[\vec{\mu}\times\vec{B}\right]\qquad \vec{\mu}(t+dt)=\vec{\mu}(t)+\gamma\left[\vec{\mu}\times\vec{B}\right]dt$$

Взаємодія.



$$\gamma\left[\vec{\mu}\times\vec{B}\right]=\frac{d\vec{\mu}}{dt}$$

$$\frac{d\vec{\mu}}{dt}=\gamma\left[\vec{\mu}\times\vec{B}\right] \qquad \vec{\mu}(t+dt)=\vec{\mu}(t)+\gamma\left[\vec{\mu}\times\vec{B}\right]dt$$

$$\vec{\omega}_0=-\gamma\vec{B}_0$$

Взаємодія.

$$\frac{d\vec{\mu}}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]$$

$$\mu_x(t) = \mu_\perp \cos(\omega_0 t + \varphi)$$

$$\mu_y(t) = \mu_\perp \sin(\omega_0 t + \varphi)$$

$$\mu_z(t) = \mu_z^{(0)} = \mu_\parallel \neq f(t)$$

$$\mu_\perp = \sqrt{\left(\mu_x^{(0)}\right)^2 + \left(\mu_y^{(0)}\right)^2}$$

Квантово-механічна взаємодія.

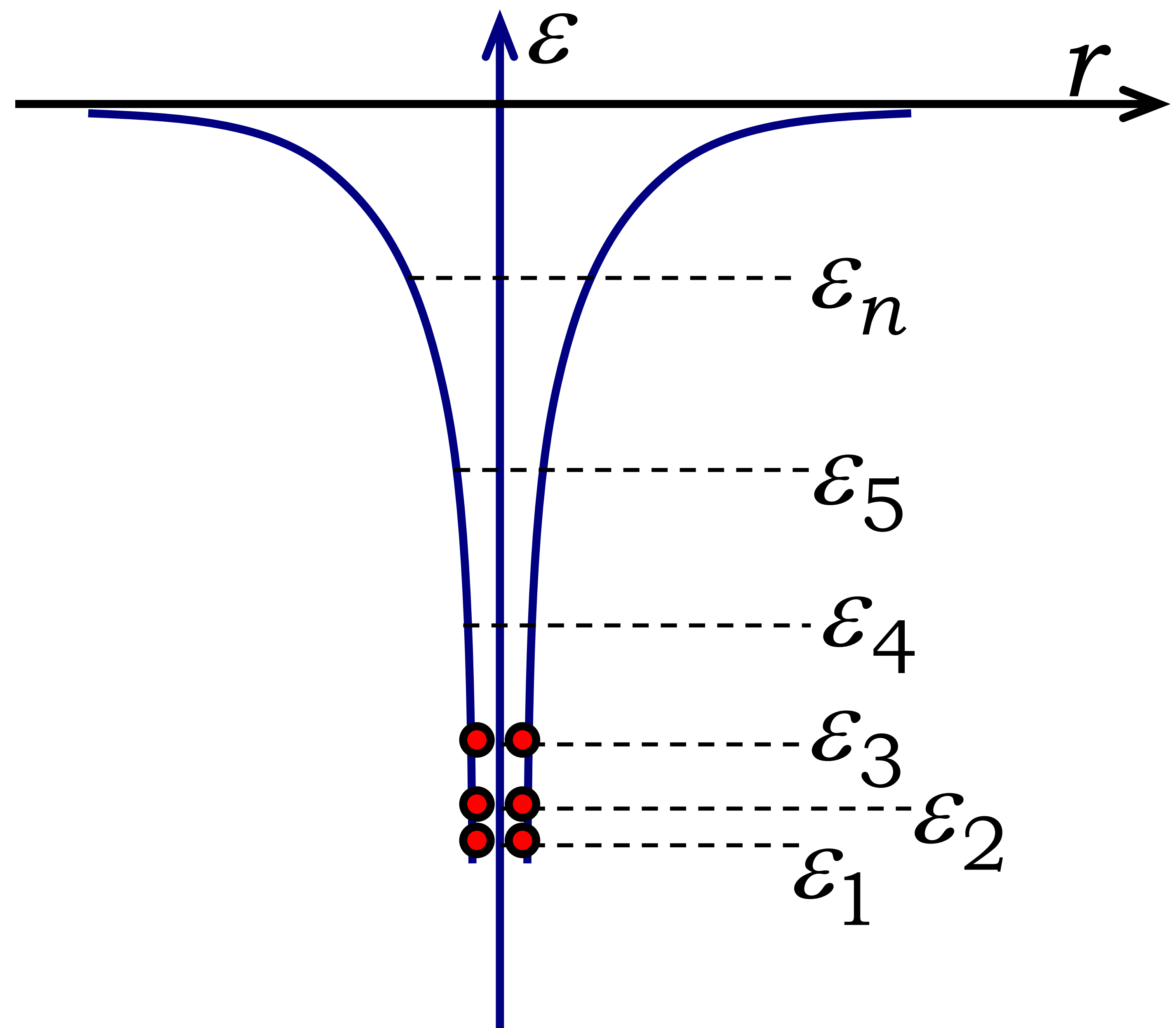
n, l, l_m

Головне, орбітальне, магнітне квантові числа

$\left[0, \infty\right)$

$\left[0, n\right]$

$\left[-l, l\right]$



Квантово-механічна взаємодія.

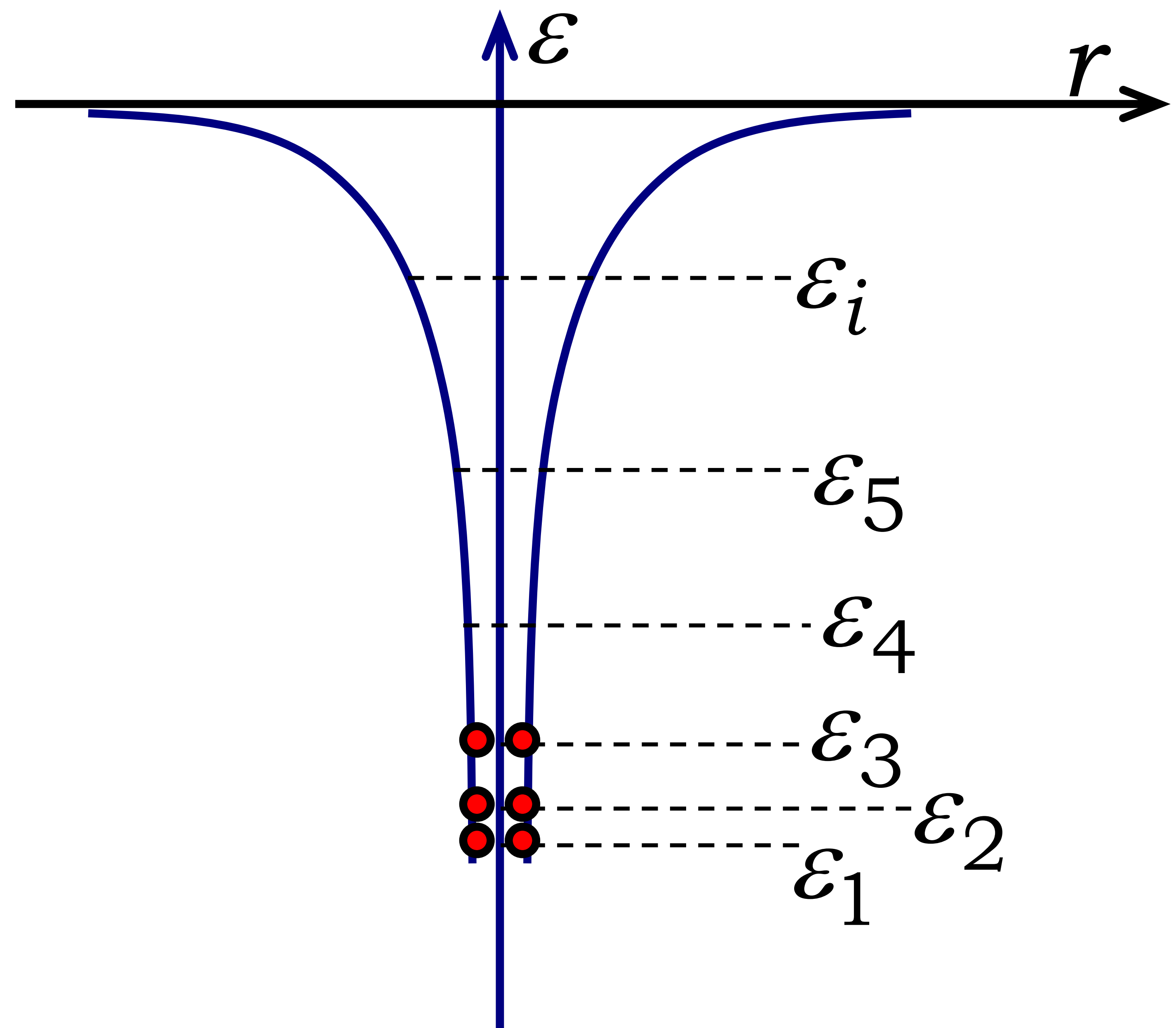
n, l, m_l індекси, які приписуються стану квантової системи для ідентифікації

Головне, орбітальне, магнітне квантові числа

$$\left[0,\infty\right)$$

$$\left[0,n\right]$$

$$\left[-l,l\right]$$



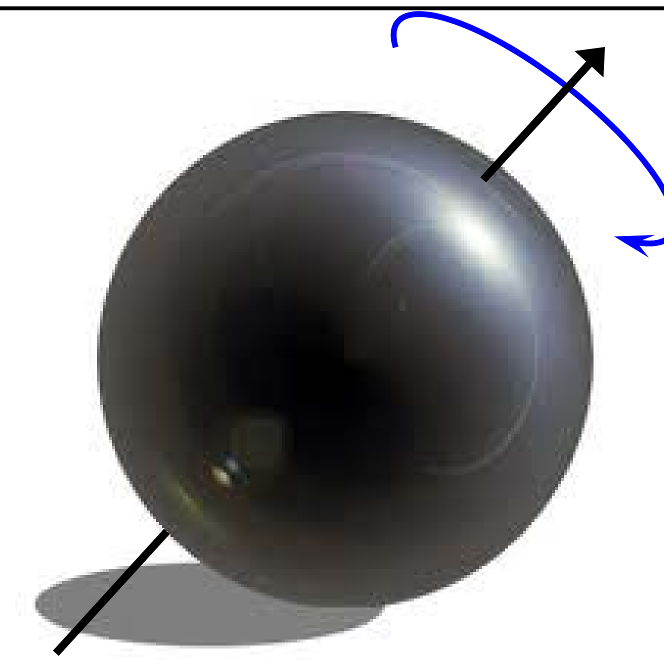
$$|J| = \hbar\sqrt{l(l+1)}$$

$$J_z = \hbar m_l$$

Квантово-механічна взаємодія.

s – спінове квантове число

m_s – магнітне спінове квантове число



$$s = \frac{1}{2} \qquad [-s, s]$$

$$|J_s| = \hbar\sqrt{s(s+1)} = \frac{\sqrt{3}\hbar}{2}$$

$$J_{sz} = m_s\hbar = \pm\frac{\hbar}{2}$$

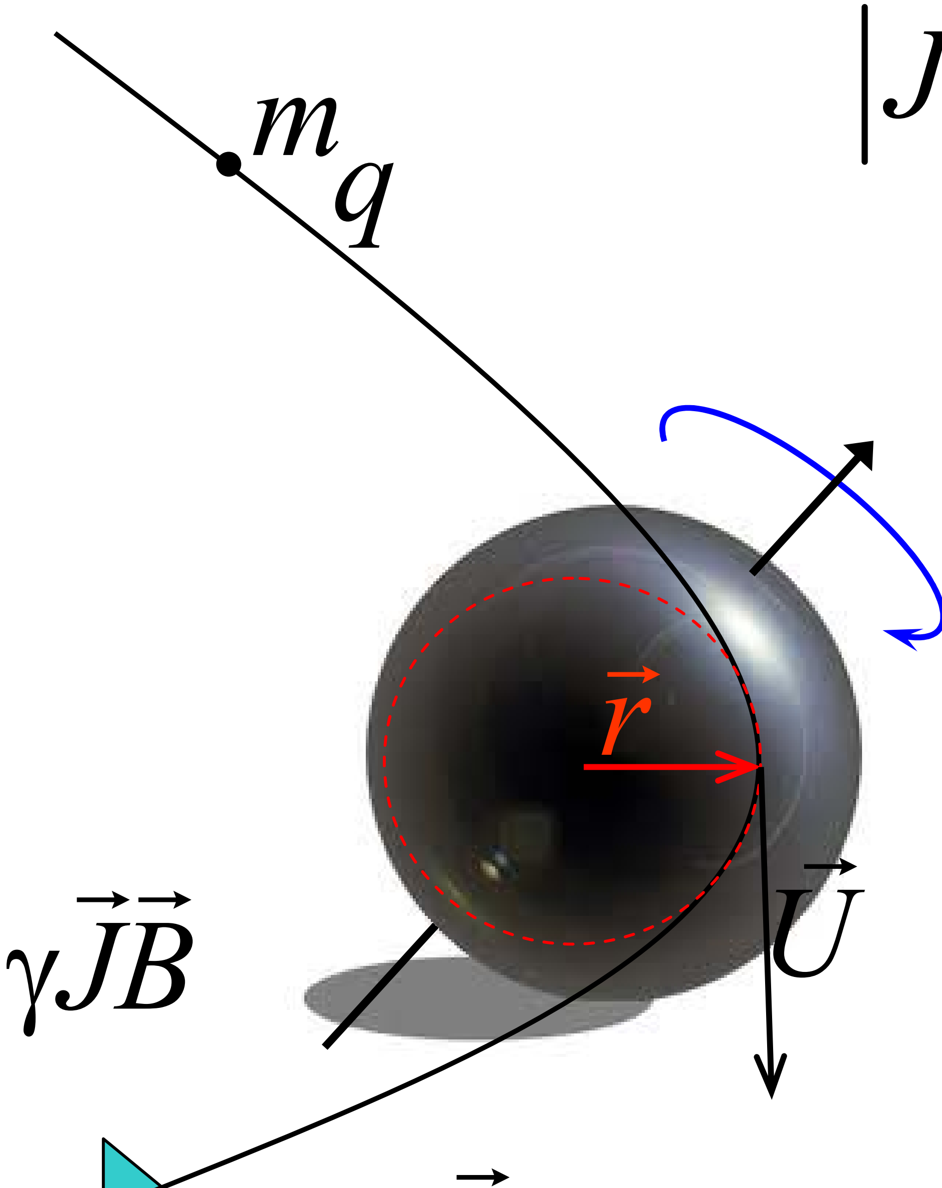
ЯМР.

$$\left|J_s\right| = \hbar\sqrt{s(s+1)} = \frac{\sqrt{3}\hbar}{2}$$

$$\vec{J} = \left[\vec{r} \times m\vec{U}\right]$$

$$\vec{\mu} = \frac{1}{2c}\left[\vec{r} \times q\vec{U}\right]$$

$$\vec{\mu} = \gamma\vec{J}$$



$\odot\ \vec{B}$

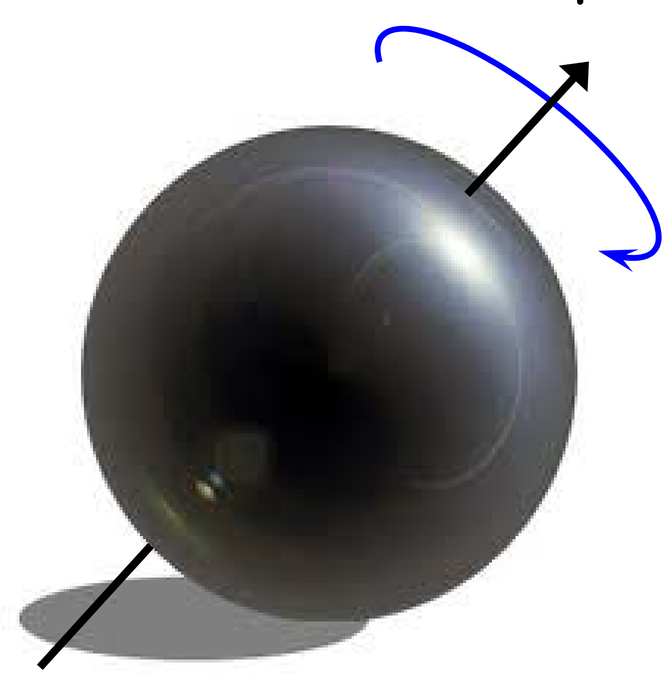
$$\varepsilon = -\vec{\mu}\vec{B} = -\gamma\vec{J}\vec{B}$$

$$\vec{M} = \left[\vec{\mu} \times \vec{B}\right] \Rightarrow \frac{d\vec{J}}{dt} = \vec{M} = \left[\vec{\mu} \times \vec{B}\right]$$

ЯМР.

$$\gamma = \frac{g|e|}{2mc} = g\gamma_0$$

$$|J_s| = \hbar\sqrt{s(s+1)} = \frac{\sqrt{3}\hbar}{2}$$



$\odot\ \vec{B}$

$$\varepsilon = -\vec{\mu}\vec{B} = -\gamma\vec{J}\vec{B}$$

$$\vec{\mu} = \gamma\vec{J}$$

$$\vec{M} = \left[\vec{\mu}\times\vec{B}\right] \Rightarrow \frac{d\vec{J}}{dt} = \vec{M} = \left[\vec{\mu}\times\vec{B}\right]$$

Взаємодія.

$$\frac{d\vec{\mu}}{dt} = \gamma\left[\vec{\mu} \times \vec{B}\right]$$

$$\mu_x(t) = \mu_x^{(0)} \cos\omega_0 t + \mu_y^{(0)} \sin\omega_0 t$$

$$\mu_y(t) = \mu_y^{(0)} \cos\omega_0 t - \mu_x^{(0)} \sin\omega_0 t$$

$$\mu_z(t) = \mu_z^{(0)} = \mu_{\parallel} \neq f(t)$$

$$\omega_0 = -\gamma B_0$$

Взаємодія.

$$\frac{d\vec{\mu}}{dt} = \gamma\left[\vec{\mu} \times \vec{B}\right]$$

$$\mu_x(t) = \mu_\perp \cos(\omega_0 t + \varphi)$$

$$\mu_y(t) = \mu_\perp \sin(\omega_0 t + \varphi)$$

$$\mu_z(t) = \mu_z^{(0)} = \mu_\parallel \neq f(t)$$

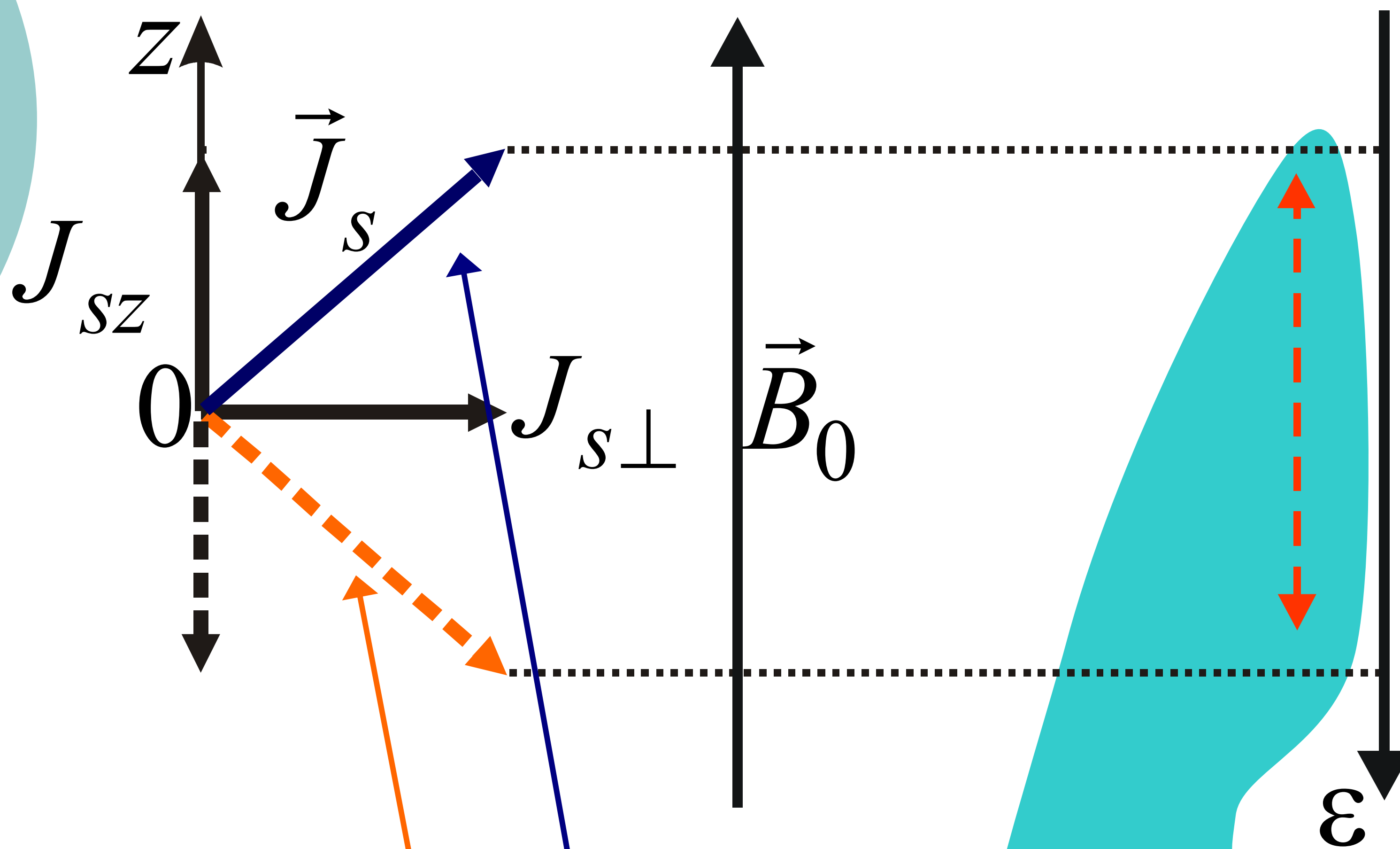
$$\mu_\perp = \sqrt{\left(\mu_x^{(0)}\right)^2 + \left(\mu_y^{(0)}\right)^2}$$

ЯМР.

$$s = \frac{1}{2} \qquad m_s = \left[-\frac{1}{2}, \frac{1}{2} \right]$$

$$\varepsilon = -\gamma \hbar m_s B_0 = \pm \frac{\gamma \hbar B_0}{2}$$

ЯМР.



$$\varepsilon_1 = -\frac{\gamma\hbar B_0}{2}$$

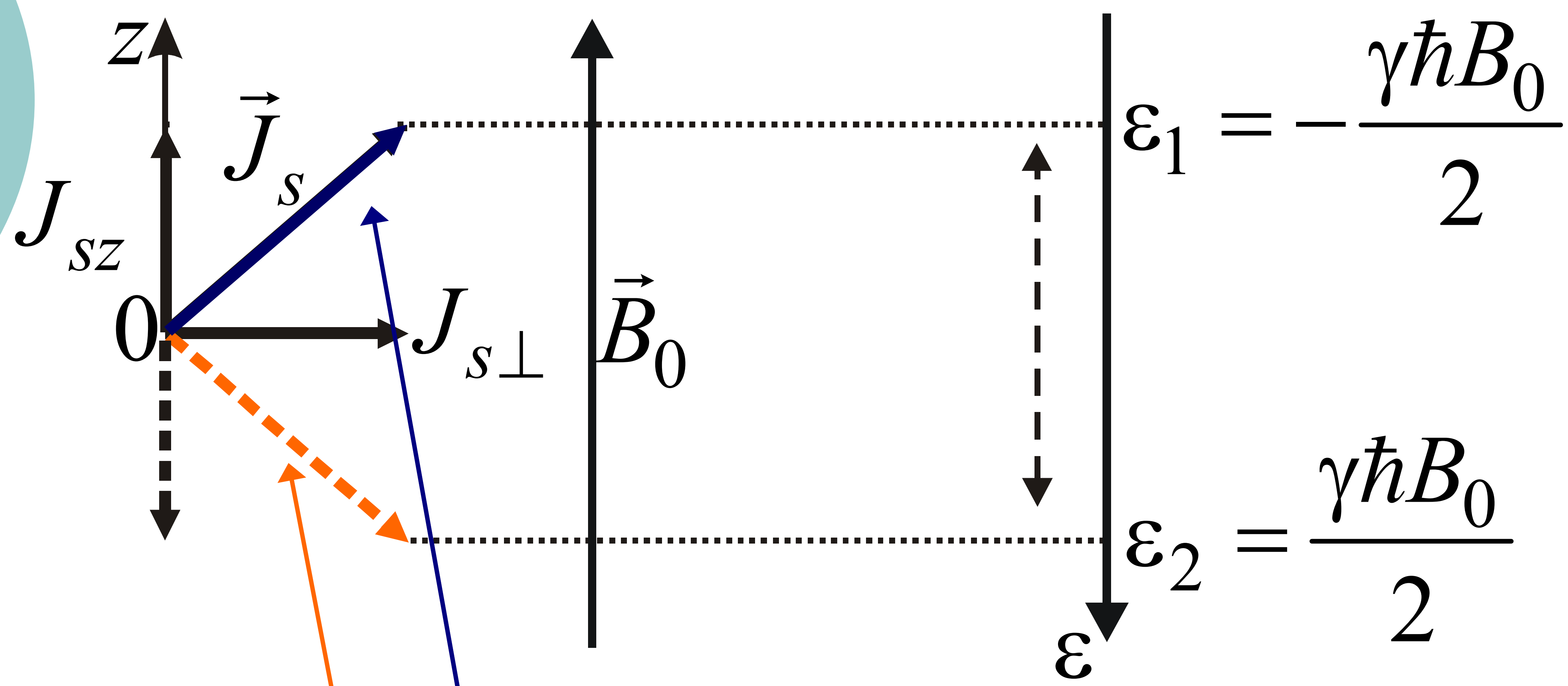
$$\varepsilon_2 = \frac{\gamma\hbar B_0}{2}$$

$$m_s = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$h\nu = \varepsilon_2 - \varepsilon_2 = \gamma\hbar B_0$$

$$2\pi\nu = \omega = \gamma B_0 \equiv \omega_0$$

ЯМР (мікро→макро перехід).



$$m_s = \left[-\frac{1}{2}, \frac{1}{2} \right]$$

ЯМР (мікро→макро перехід).

$$A^{(MAKPO)} = \sum_{i=1}^{N} A_i^{(MIKPO)}$$

$$\vec{B}_0 = 0 \Rightarrow \vec{M}_0 = 0$$

$$\vec{B}_0 \neq 0 \Rightarrow \begin{cases} N_\uparrow \\ N_\downarrow \end{cases} \qquad N = N_\uparrow + N_\downarrow$$

$$\frac{N_\downarrow}{N_\uparrow} = \exp\left(-\frac{\varepsilon}{kT}\right) = \exp\left(-\frac{\gamma\hbar B_0}{kT}\right)$$

ЯМР (мікро→макро перехід).

$$\varepsilon = \frac{2,67 \cdot 10^{8} \cdot 6,62 \cdot 10^{-34}}{2\pi} \approx$$

$$\approx 2,8 \cdot 10^{-26}\ \text{Дж} \approx 1,76 \cdot 10^{-7}\ \text{еВ}$$

ЯМР (мікро⟶макро перехід).

$$\vec{B}_0 \neq 0 \Rightarrow \begin{cases} N_\uparrow \\ N_\downarrow \end{cases} \qquad N = N_\uparrow + N_\downarrow$$

$$\frac{N_\downarrow}{N_\uparrow} = \exp\left(-\frac{\varepsilon}{kT}\right) = \exp\left(-\frac{\gamma\hbar B_0}{kT}\right)$$

$$\frac{N_\downarrow}{N_\uparrow} = \exp\left(-\frac{2,67\cdot 10^{8}\cdot 6,62\cdot 10^{-34}}{2\pi\cdot 1,38\cdot 10^{-23}\cdot 300}\right) \approx \exp\left(-6,8\cdot 10^{-6}\right)$$

$$\eta = \frac{N_\downarrow}{N_\uparrow} \approx 0,9999932$$

ЯМР (мікро→макро перехід).

$$\eta = \frac{N_{\downarrow}}{N_{\uparrow}} \approx 0,9999932$$

$$N_{\uparrow} = \frac{N}{1+\eta} = 0,5000017 \cdot N$$

$$N_{\downarrow} = \frac{\eta}{\eta+1} N = 0,4999983 \cdot N$$

$$N_{\uparrow} - N_{\downarrow} = \frac{1-\eta}{\eta+1} N = 0,0000034 \cdot N$$

ЯМР (мікро→макро перехід).

H_2O

$$N = N_A \cdot \nu = N_A \frac{m}{M} = N_A \frac{\rho V}{M}$$

$$N = \frac{6,022 \cdot 10^{23} \cdot 10^{3} \cdot 10^{-6}}{18 \cdot 10^{3}} = 3,35 \cdot 10^{22}$$

ЯМР (мікро→макро перехід).

$$H_2O \qquad N = \frac{6,022 \cdot 10^{23} \cdot 10^{3} \cdot 10^{-6}}{18 \cdot 10^{3}} = 3,35 \cdot 10^{22}$$

$$N_{\uparrow} = 1,6750057 \cdot 10^{22}$$

$$N_{\downarrow} = 1,6749943 \cdot 10^{22}$$

$$N_{\uparrow} - N_{\downarrow} = 1,139 \cdot 10^{17}$$

$$M_0 = \frac{N\hbar^2\gamma^2 J\left(J+1\right)B_0}{3kT}$$

Рівняння Блоха.

$$\frac{d\vec{\mu}}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]$$

$$\begin{cases} \dfrac{d\mu_x}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_x = \gamma\left(\mu_y B_z - \mu_z B_y\right) \\ \dfrac{d\mu_y}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_y = \gamma\left(\mu_z B_x - \mu_x B_z\right) \\ \dfrac{d\mu_z}{dt} = \gamma\left[\vec{\mu}\times\vec{B}\right]_z = \gamma\left(\mu_x B_y - \mu_y B_x\right) \end{cases}$$

Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] - \frac{M_x\vec{i} + M_y\vec{j}}{T_2} - \frac{M_z - M_0}{T_1}\vec{k}$$

$$\begin{cases} \dfrac{dM_x}{dt} = \gamma\left(M_y B_z - M_z B_y\right) - \dfrac{M_x}{T_2} \\ \dfrac{dM_y}{dt} = \gamma\left(M_z B_x - M_x B_z\right) - \dfrac{M_y}{T_2} \\ \dfrac{dM_z}{dt} = \gamma\left(M_x B_y - M_y B_x\right) - \dfrac{M_z - M_0}{T_1} \end{cases}$$

Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right]$$

$$\begin{cases} \dfrac{dM_x}{dt} = \gamma\left(M_y B_z - M_z B_y\right) \\ \dfrac{dM_y}{dt} = \gamma\left(M_z B_x - M_x B_z\right) \\ \dfrac{dM_z}{dt} = \gamma\left(M_x B_y - M_y B_x\right) \end{cases}$$

Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}_0\right] \quad \vec{B}_0 = \left\{B_x, B_y, B_z\right\} = \left\{0, 0, B_0\right\}$$

$$\begin{cases} \dfrac{dM_x}{dt} = \gamma M_y B_0 \\ \dfrac{dM_y}{dt} = -\gamma M_x B_0 \\ \dfrac{dM_z}{dt} = 0 \end{cases}$$

Розв'язки рівняння Блоха.

$$M_x(t) = M_x^{(0)} \cos \omega_0 t + M_y^{(0)} \sin \omega_0 t$$

$$M_y(t) = M_y^{(0)} \cos \omega_0 t - M_x^{(0)} \sin \omega_0 t$$

$$M_z(t) = M_z^{(0)} = M_{\parallel} \neq f(t)$$

$$\omega_0 = -\gamma B_0$$

Розв'язки рівняння Блоха.

$$M_x(t) = M_\perp \cos(\omega_0 t + \varphi)$$

$$M_y(t) = M_\perp \sin(\omega_0 t + \varphi)$$

$$M_z(t) = M_z^{(0)} = M_\parallel \neq f(t)$$

$$M_\perp = \sqrt{\left(M_x^{(0)}\right)^2 + \left(M_y^{(0)}\right)^2}$$

Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] - \frac{M_x\vec{i} + M_y\vec{j}}{T_2} - \frac{M_z - M_0}{T_1}\vec{k}$$

$$\begin{cases} \dfrac{dM_x}{dt} = \gamma M_y B_0 - \dfrac{M_x}{T_2} \\ \dfrac{dM_y}{dt} = -\gamma M_x B_0 - \dfrac{M_y}{T_2} \\ \dfrac{dM_z}{dt} = \dfrac{M_0 - M_z}{T_1} \end{cases}$$

Розв'язки рівняння Блоха.

$$M_x(t) = \left(M_x^{(0)} \cos \omega_0 t + M_y^{(0)} \sin \omega_0 t \right) \cdot \exp\left(-\frac{t}{T_2} \right)$$

$$M_y(t) = \left(M_y^{(0)} \cos \omega_0 t - M_x^{(0)} \sin \omega_0 t \right) \cdot \exp\left(-\frac{t}{T_2} \right)$$

$$M_z(t) = M_z^{(0)} \cdot \exp\left(-\frac{t}{T_1} \right) + M_0 \cdot \left[1 - \exp\left(-\frac{t}{T_1} \right) \right]$$

$$\omega_0 = -\gamma B_0$$

Розв'язки рівняння Блоха.

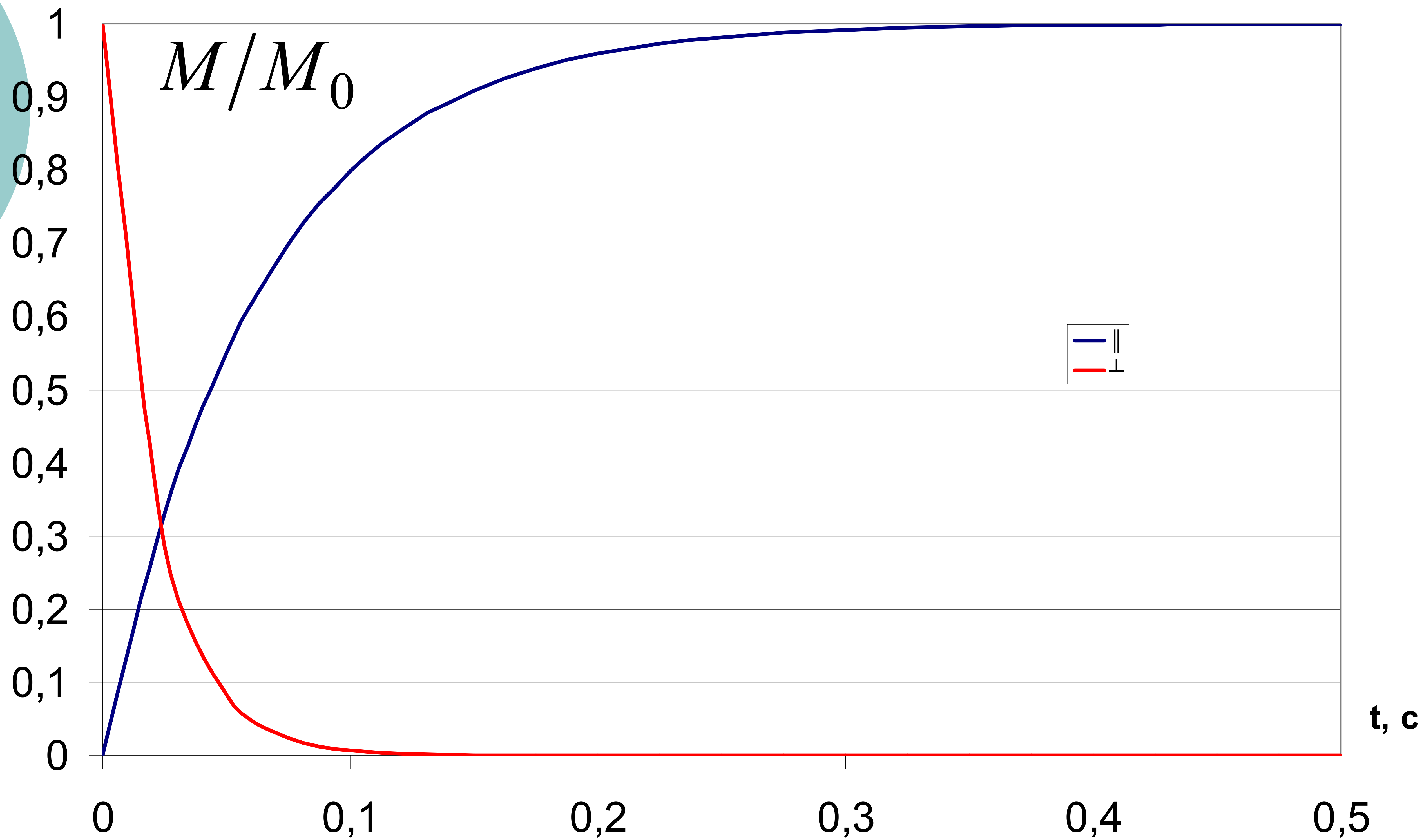
$$M_{\perp}^{(0)} = M_x^{(0)} + iM_y^{(0)}$$

$$M_{\perp}(t) = M_{\perp}^{(0)} \exp\left(i\omega_0 t - \frac{t}{T_2} \right)$$

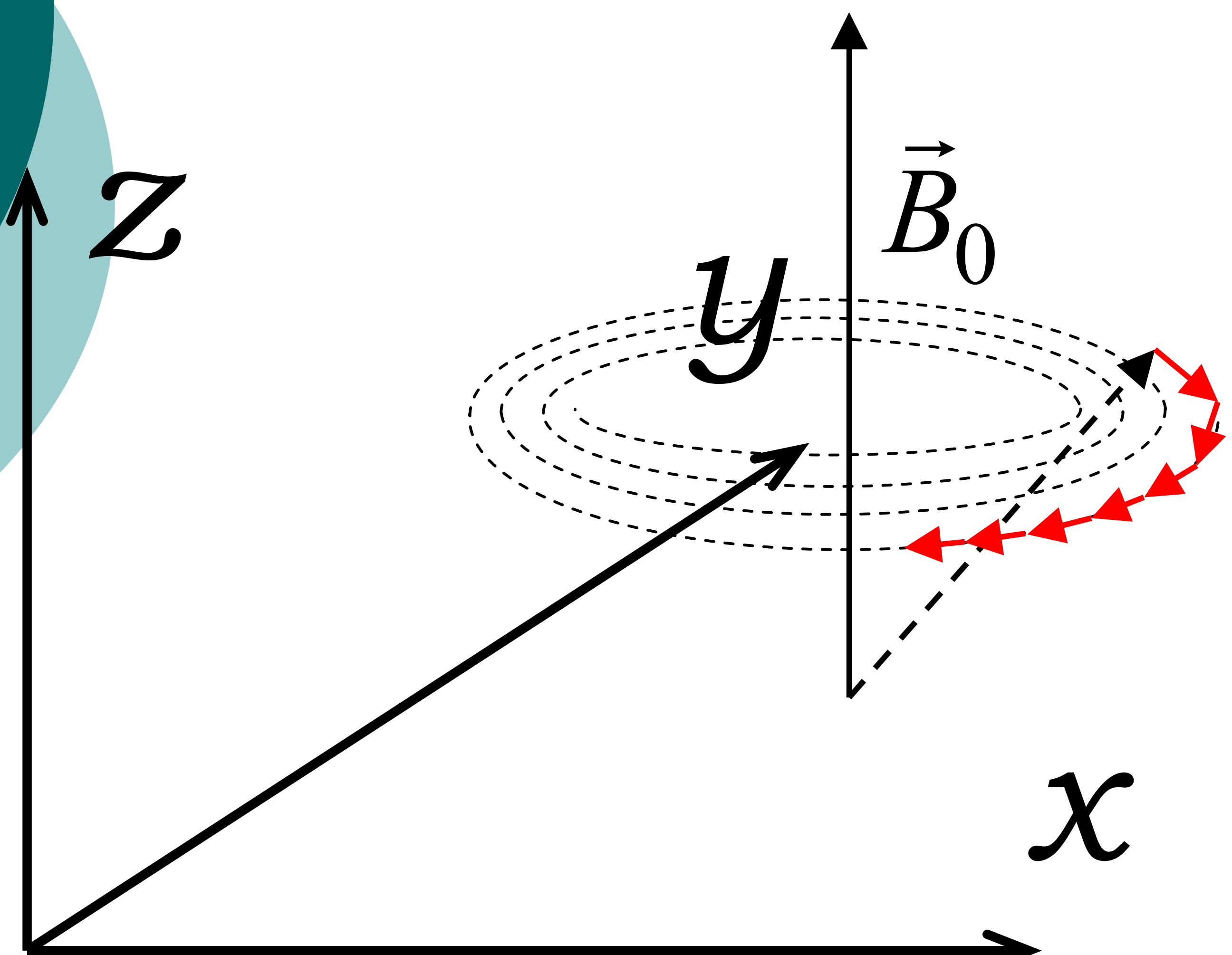
$$M_{\parallel}(t) = M_z^{(0)} \cdot \exp\left(-\frac{t}{T_1} \right) + M_0 \cdot \left[1 - \exp\left(-\frac{t}{T_1} \right) \right]$$

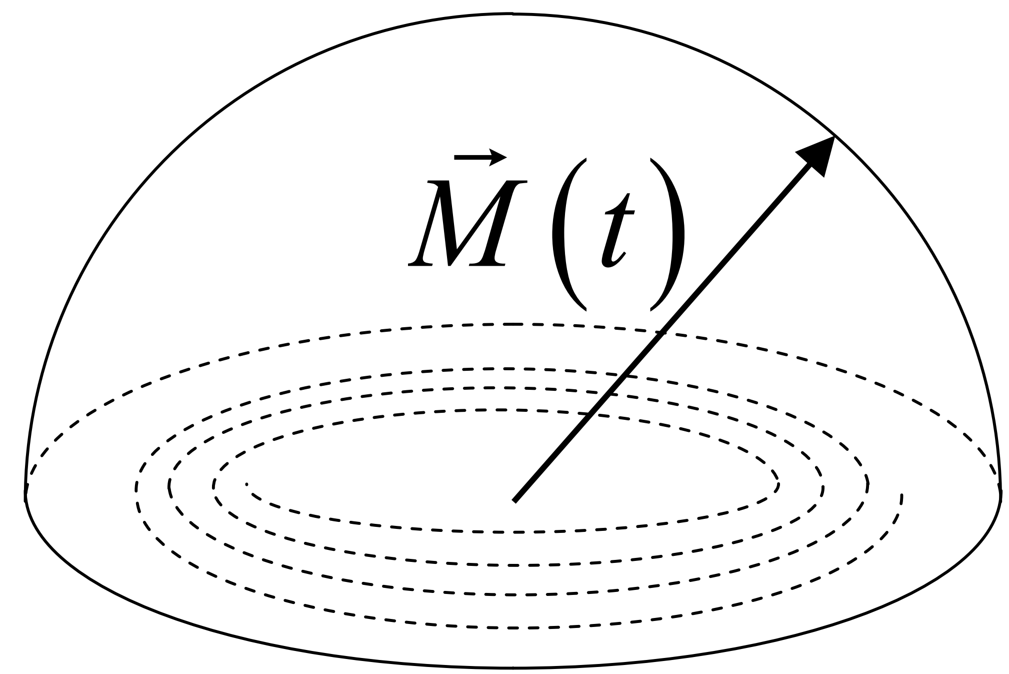
$$\omega_0 = -\gamma B_0$$

Розв’язки рівняння Блоха.



Взаємодія.





$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}_0\right] - \frac{M_x\vec{i} + M_y\vec{j}}{T_2} - \frac{M_z - M_0}{T_1}\vec{k}$$

Поле градієнтів.

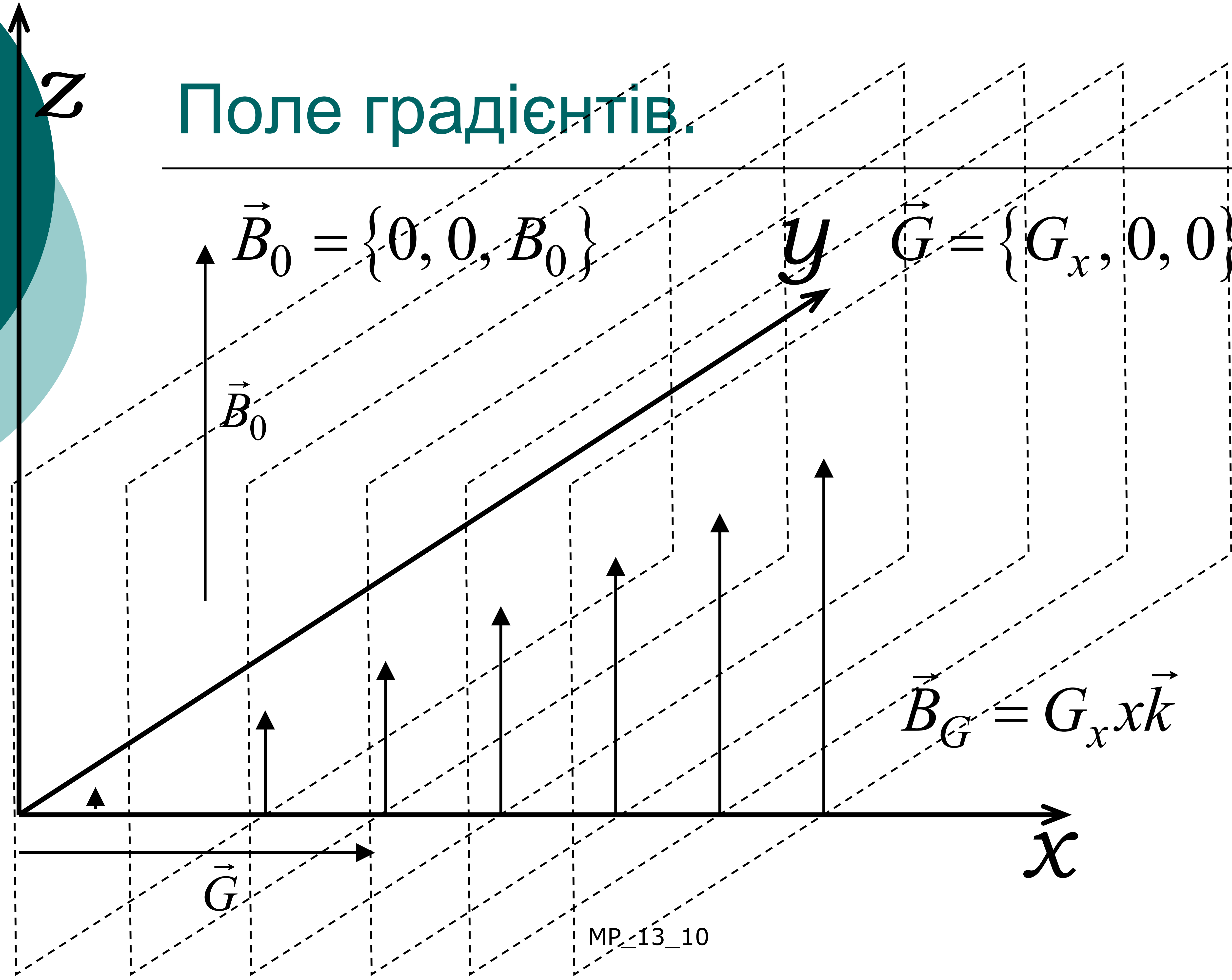
$$\vec{B}_G = \{0, 0, B_G\} \qquad \vec{B}_G = B_G\vec{k}$$

$$B_G = \vec{G}\vec{r} \qquad \vec{G} = \{G_x, G_y, G_z\} \qquad \vec{r} = \{x, y, z\}$$

$$\vec{B}_G = \left(G_x x + G_y y + G_z z\right)\vec{k}$$

$$\vec{G} = \{G_x, 0, 0\} \qquad \vec{B}_G = G_x x\vec{k}$$

Поле градієнтів.



Поле градієнтів.

$$\vec{B}_G = \{0,\, 0,\, B_G\} \qquad \vec{B}_G = B_G\vec{k}$$

$$B_G = \vec{G}\vec{r} \qquad \vec{G} = \{G_x,\, G_y,\, G_z\} \qquad \vec{r} = \{x,\, y,\, z\}$$

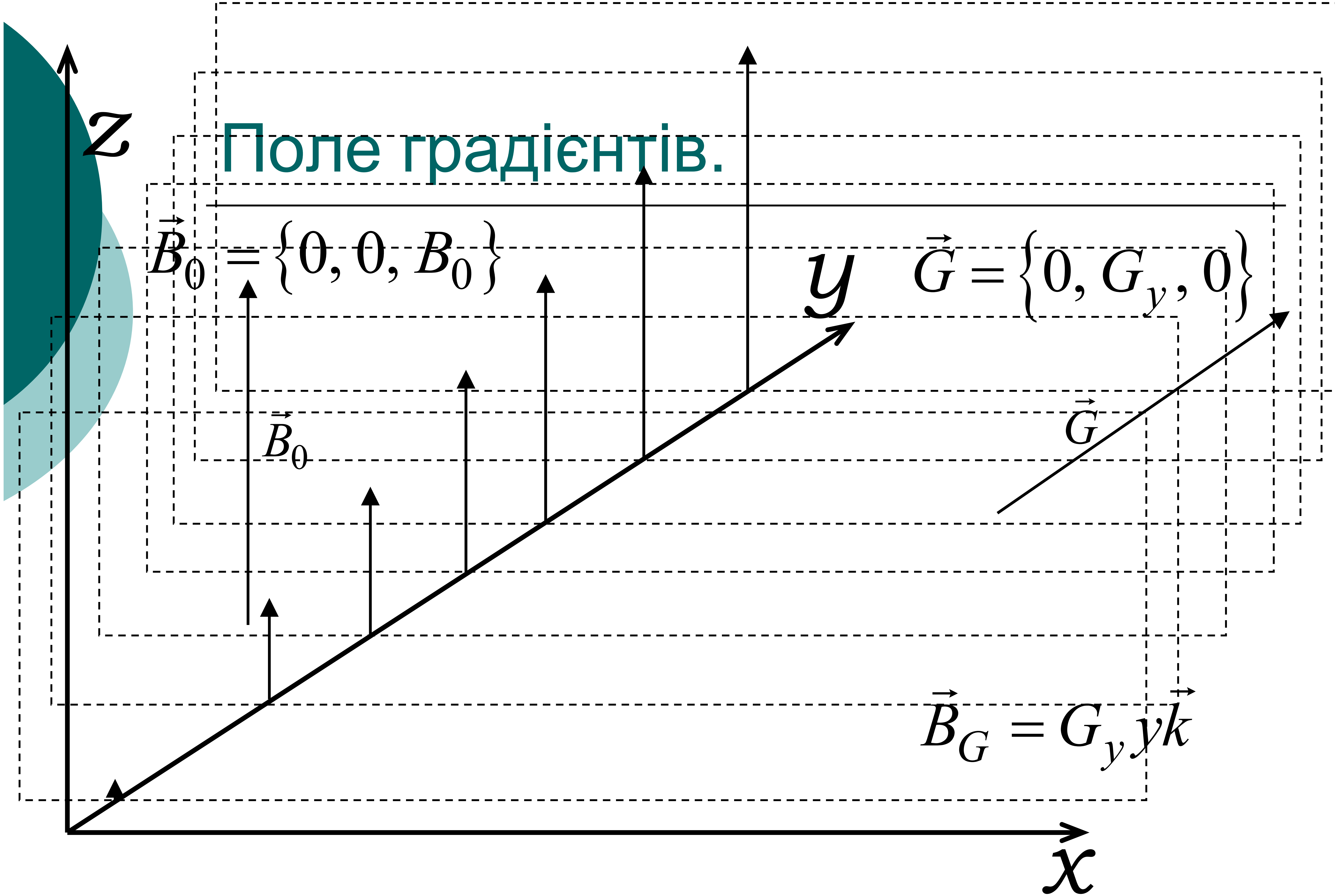
$$\vec{B}_G = \left(G_x x + G_y y + G_z z\right)\vec{k}$$

$$\vec{G} = \{G_x,\, 0,\, 0\} \qquad \vec{B}_G = G_x x\vec{k}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_G = \{0,\, 0,\, B_0 + B_G\} = \{0,\, 0,\, B_0 + G_x x\}$$

$$\vec{G} = \{0,\, G_y,\, 0\} \qquad \vec{B}_G = G_y y\vec{k}$$

Поле градієнтів.



Поле градієнтів.

$$\vec{B}_G = \{0, 0, B_G\} \qquad \vec{B}_G = B_G \vec{k}$$

$$B_G = \vec{G}\vec{r} \qquad \vec{G} = \{G_x, G_y, G_z\} \qquad \vec{r} = \{x, y, z\}$$

$$\vec{B}_G = \left(G_x x + G_y y + G_z z\right)\vec{k}$$

$$\vec{G} = \{G_x, 0, 0\} \qquad \vec{B}_G = G_x x \vec{k}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_G = \{0, 0, B_0 + B_G\} = \{0, 0, B_0 + G_x x\}$$

$$\vec{G} = \{0, G_y, 0\} \qquad \vec{B}_G = G_y y \vec{k}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_G = \{0, 0, B_0 + B_G\} = \{0, 0, B_0 + G_y y\}$$

Поле градієнтів.

$$\vec{B}_G = \{0, 0, B_G\} \qquad \vec{B}_G = B_G \vec{k}$$

$$B_G = \vec{G}\vec{r} \qquad \vec{G} = \{G_x, G_y, G_z\} \qquad \vec{r} = \{x, y, z\}$$

$$\vec{B}_G = \left(G_x x + G_y y + G_z z\right)\vec{k}$$

$$\vec{G} = \{G_x, 0, 0\} \qquad \vec{B}_G = G_x x \vec{k}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_G = \{0, 0, B_0 + B_G\} = \{0, 0, B_0 + G_x x\}$$

$$\vec{G} = \{0, G_y, 0\} \qquad \vec{B}_G = G_y y \vec{k}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_G = \{0, 0, B_0 + B_G\} = \{0, 0, B_0 + G_y y\}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_G = \{0, 0, B_0 + B_G\} = \{0, 0, B_0 + G_z z\}$$

Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] - \frac{M_x\vec{i} + M_y\vec{j}}{T_2} - \frac{M_z - M_0}{T_1}\vec{k}$$

$$\begin{cases} \dfrac{dM_x}{dt} = \gamma M_y B - \dfrac{M_x}{T_2} \\ \dfrac{dM_y}{dt} = -\gamma M_x B - \dfrac{M_y}{T_2} \\ \dfrac{dM_z}{dt} = \dfrac{M_0 - M_z}{T_1} \end{cases}$$

Розв'язки рівняння Блоха.

$$M_{\perp}(t) = M_{\perp}^{(0)} \exp\left(i\omega t - \frac{t}{T_2}\right)$$

$$M_{\parallel}(t) = M_z^{(0)} \cdot \exp\left(-\frac{t}{T_1}\right) + M_0 \cdot \left[1 - \exp\left(-\frac{t}{T_1}\right)\right]$$

$$\omega = -\gamma\left(B_0 + G_x x + G_y y + G_z z\right) =$$

$$= \omega_0 + \gamma\left(G_x x + G_y y + G_z z\right) = \omega_0 + f\left(\vec{r}\right)$$

Просторове кодування.

$$M_{\perp}(t) = M_{\perp}^{(0)} \exp\left(i\omega t - \frac{t}{T_2} \right)$$

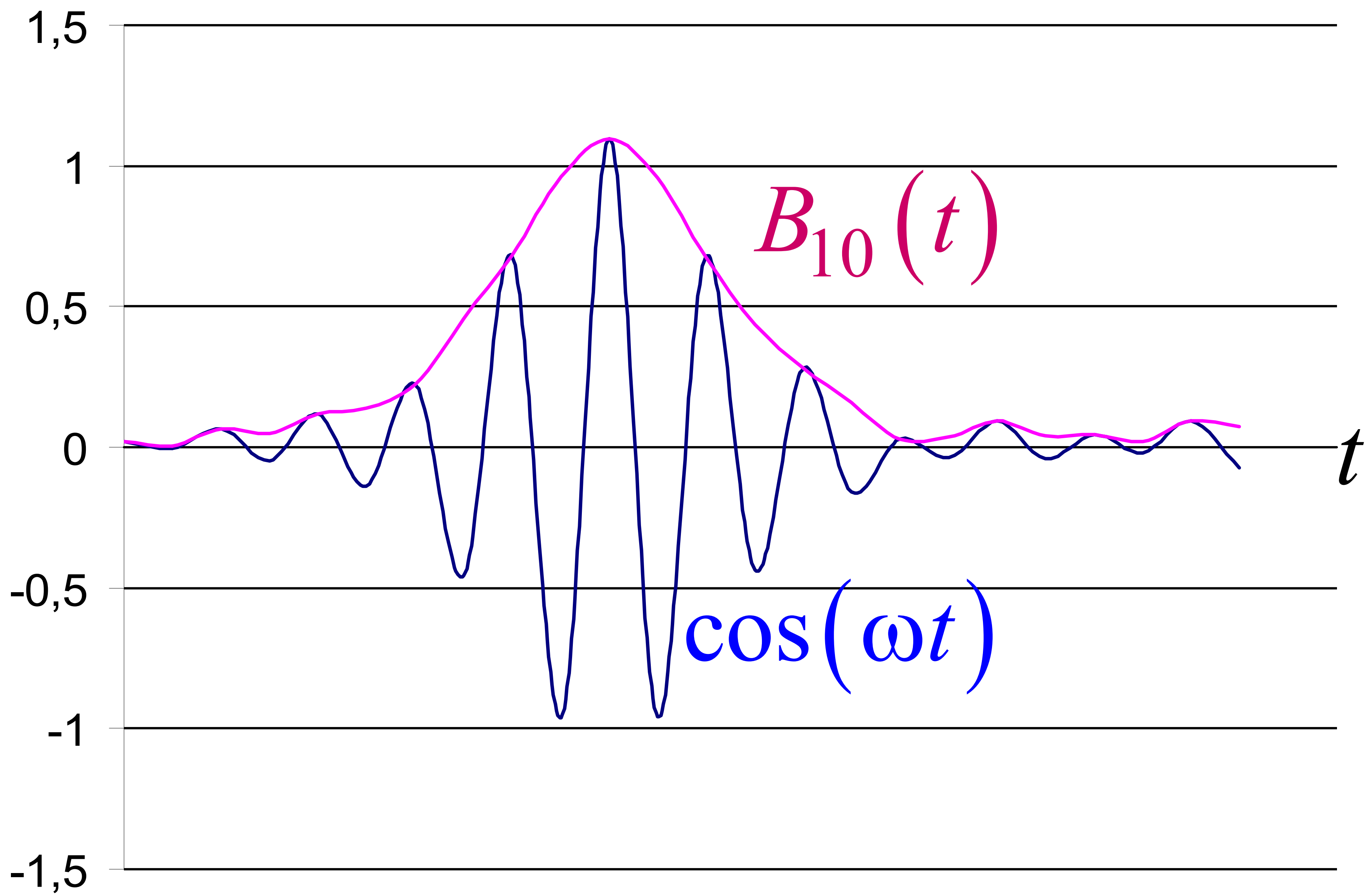
$$M_{\parallel}(t) = M_z^{(0)} \cdot \exp\left(-\frac{t}{T_1} \right) + M_0 \cdot \left[1 - \exp\left(-\frac{t}{T_1} \right) \right]$$

$$\omega = \omega_0 + f(\vec{r}) \qquad \vec{B}_G = B_G \vec{k}$$

$$B_G = \vec{G}\vec{r} \qquad \vec{G} = \{G_x, G_y, G_z\} \qquad \vec{r} = \{x, y, z\}$$

РЧ поле.

$$\vec{B}_1(t) = B_{10}(t)\cos(\omega t)\vec{i}$$

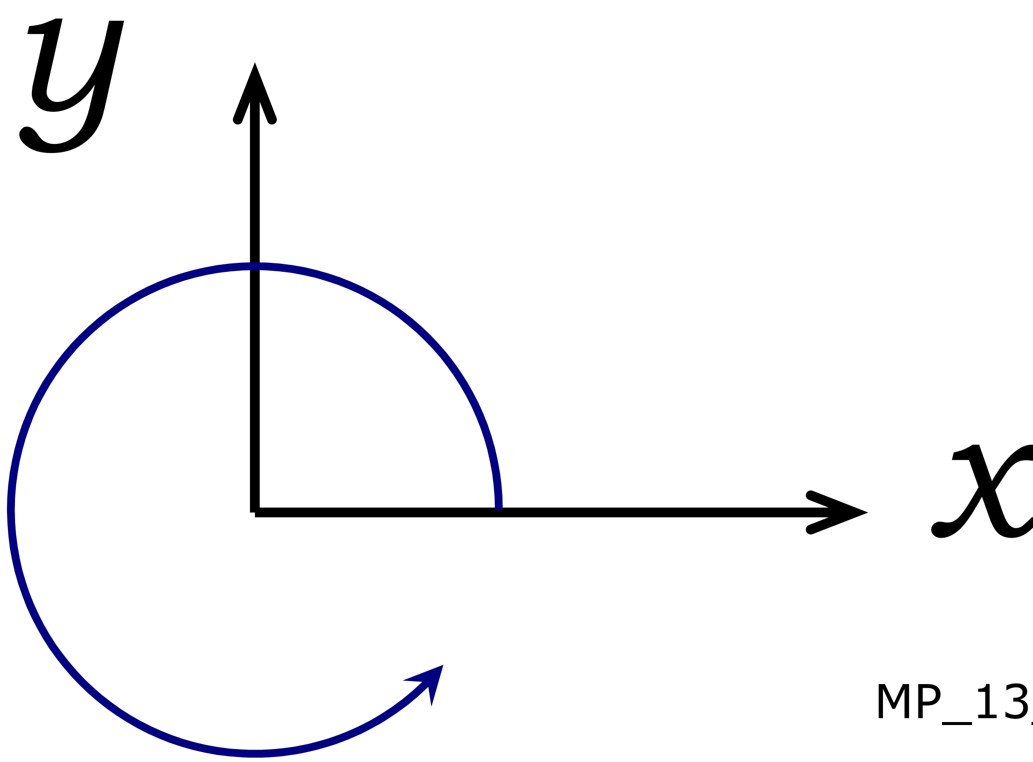


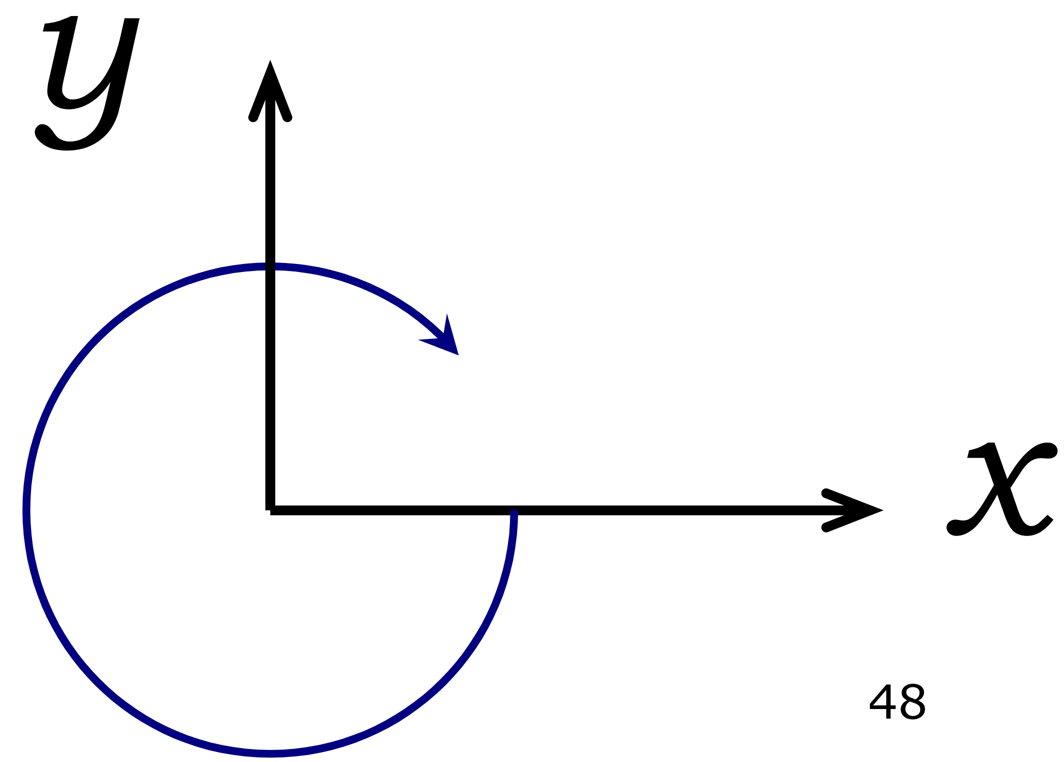
РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\cos(\omega t)\vec{i}$$

$$\vec{B}_1(t) = B_{10}(t)\left[\cos(\omega t)\vec{i} + \sin(\omega t)\vec{j}\right] +$$

$$+ B_{10}(t)\left[\cos(\omega t)\vec{i} - \sin(\omega t)\vec{j}\right]$$

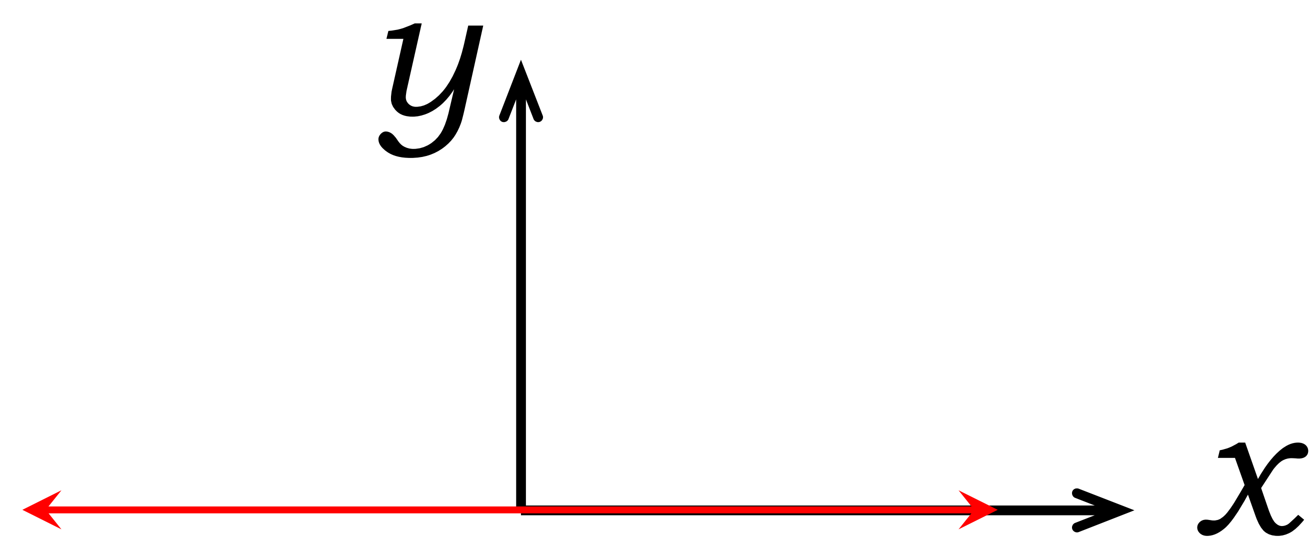




РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\cos(\omega t)\vec{i}$$

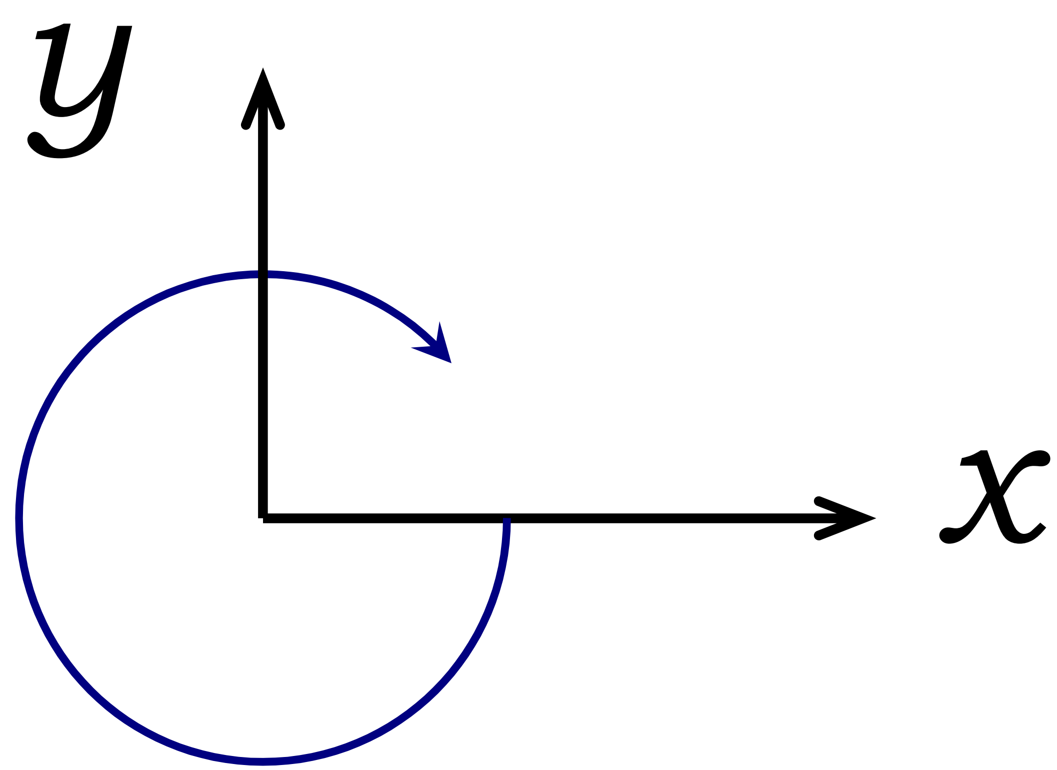
$$\vec{B}_1(t) = B_{10}(t)\left[\cos(\omega t)\vec{i} + \sin(\omega t)\vec{j}\right] +$$
$$+ B_{10}(t)\left[\cos(\omega t)\vec{i} - \sin(\omega t)\vec{j}\right]$$



РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\cos(\omega t)\vec{i}$$

$$\vec{B}_1(t) = B_{10}(t)\left[\cos(\omega t)\vec{i} - \sin(\omega t)\vec{j}\right]$$



Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\begin{cases} \dfrac{dM_x}{dt} = \gamma M_y B_0 + \gamma M_z B_{10}(t)\sin(\omega t) \\ \dfrac{dM_y}{dt} = \gamma M_z B_{10}(t)\cos(\omega t) - \gamma M_x B_0 \\ \dfrac{dM_z}{dt} = -\gamma M_x B_{10}(t)\sin(\omega t) - \gamma M_y B_{10}(t)\cos(\omega t) \end{cases}$$

Заміна.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\begin{cases} M_x = \mathrm{u}\cos(\omega t) - \mathrm{v}\sin(\omega t) \\ M_y = \mathrm{u}\sin(\omega t) + \mathrm{v}\cos(\omega t) \\ M_z = M_z \end{cases}$$

$$B_{10}(t) = \begin{cases} B_{10} = const \neq f(t), 0 \leq t \leq \tau \\ 0, t > \tau \end{cases}$$

Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\begin{cases} \dfrac{d\mathrm{u}}{dt} = \left(\gamma B_0 + \omega\right)\mathrm{v} \\ \dfrac{d\mathrm{v}}{dt} = -\left(\gamma B_0 + \omega\right)\mathrm{u} + \gamma B_{10} M_z \\ \dfrac{dM_z}{dt} = -\gamma B_0 \mathrm{v} \end{cases}$$

Умова резонансу. Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\begin{cases} \dfrac{d\mathrm{u}}{dt} = \left(\gamma B_0 + \omega\right)\mathrm{v} \\ \dfrac{d\mathrm{v}}{dt} = -\left(\gamma B_0 + \omega\right)\mathrm{u} + \gamma B_{10} M_z \\ \dfrac{dM_z}{dt} = -\gamma B_0 \mathrm{v} \end{cases}$$

$\omega = \omega_0$

Умова резонансу. Рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\begin{cases} \dfrac{d\mathrm{u}}{dt} = 0 \\ \dfrac{d\mathrm{v}}{dt} = \gamma B_{10} M_z \\ \dfrac{dM_z}{dt} = -\gamma B_0 \mathrm{v} \end{cases} \qquad \begin{cases} \dfrac{dM_x}{dt} = \gamma M_y B_0 \\ \dfrac{dM_y}{dt} = -\gamma M_x B_0 \\ \dfrac{dM_z}{dt} = 0 \end{cases}$$

$$\omega = \omega_0$$

Умова резонансу. Розв'язки рівняння Блоха.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\mathrm{v}(t) = \mathrm{v}^{(0)} \cos \omega_1 t + M_z^{(0)} \sin \omega_1 t$$

$$M_z(t) = M_z^{(0)} \cos \omega_1 t - \mathrm{v}^{(0)} \sin \omega_1 t$$

$$\mathrm{u}(t) = \mathrm{u}^{(0)} = const \neq f(t)$$

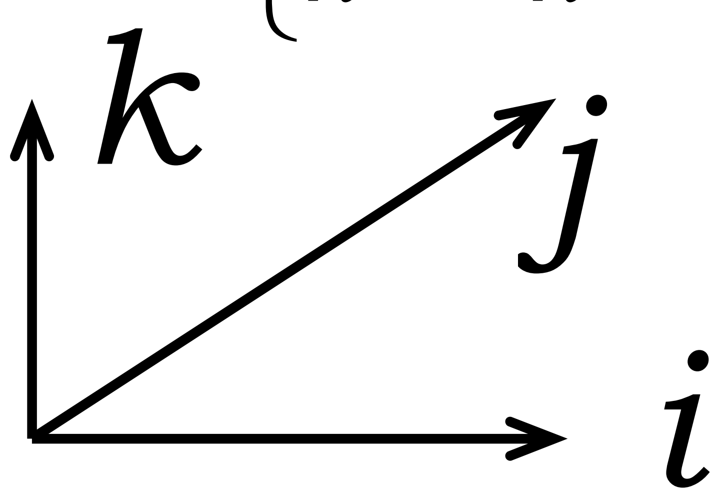
$$\omega = \omega_0$$

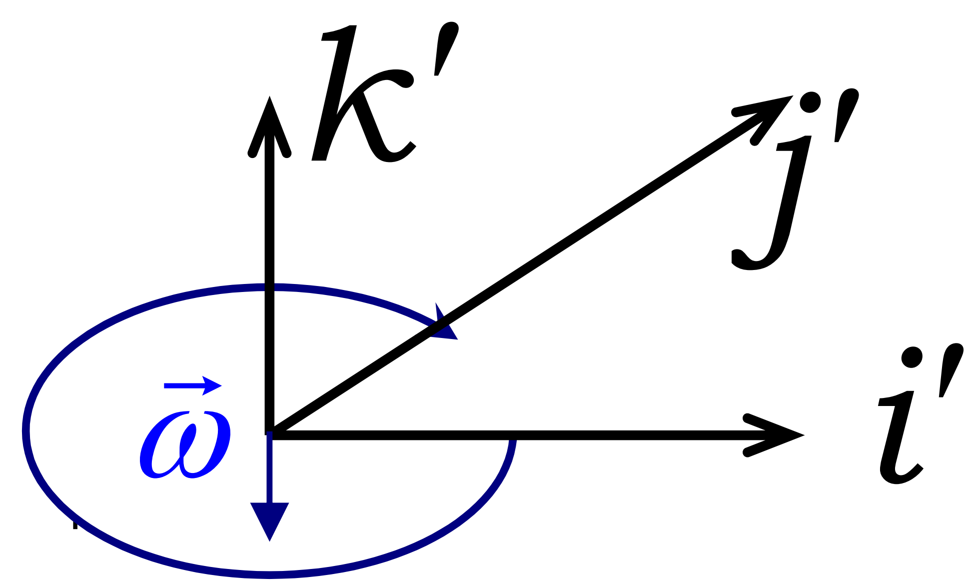
$$\omega_1 = -\gamma B_1$$

Система координат.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\begin{cases} \vec{i}' = \vec{i}\cos(\omega t) - \vec{j}\sin(\omega t) \\ \vec{j}' = -\vec{i}\sin(\omega t) + \vec{j}\cos(\omega t) \\ \vec{k}' = \vec{k} \end{cases}$$

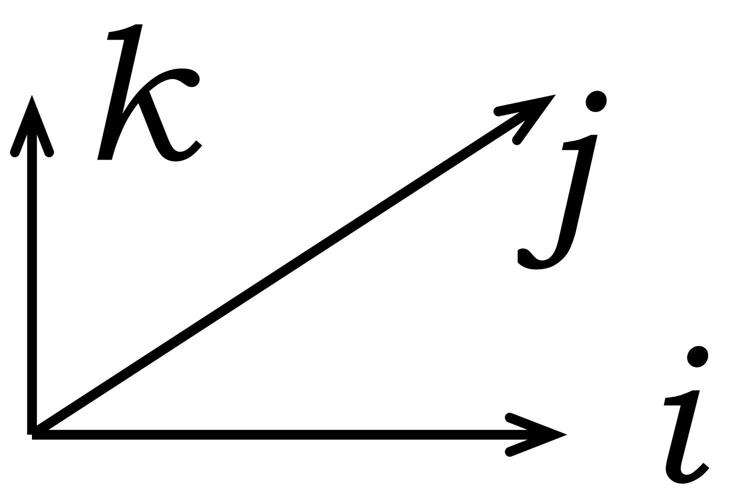


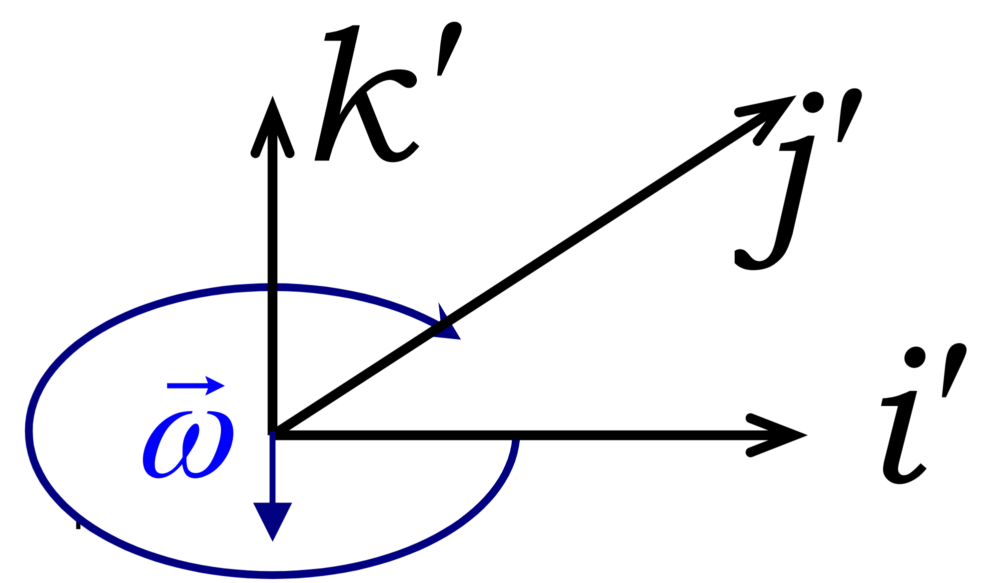


Система координат.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

$$\vec{i}'\mathrm{u}(t) + \vec{j}'\mathrm{v}(t) = \vec{i}M_x(t) + \vec{j}M_y(t)$$





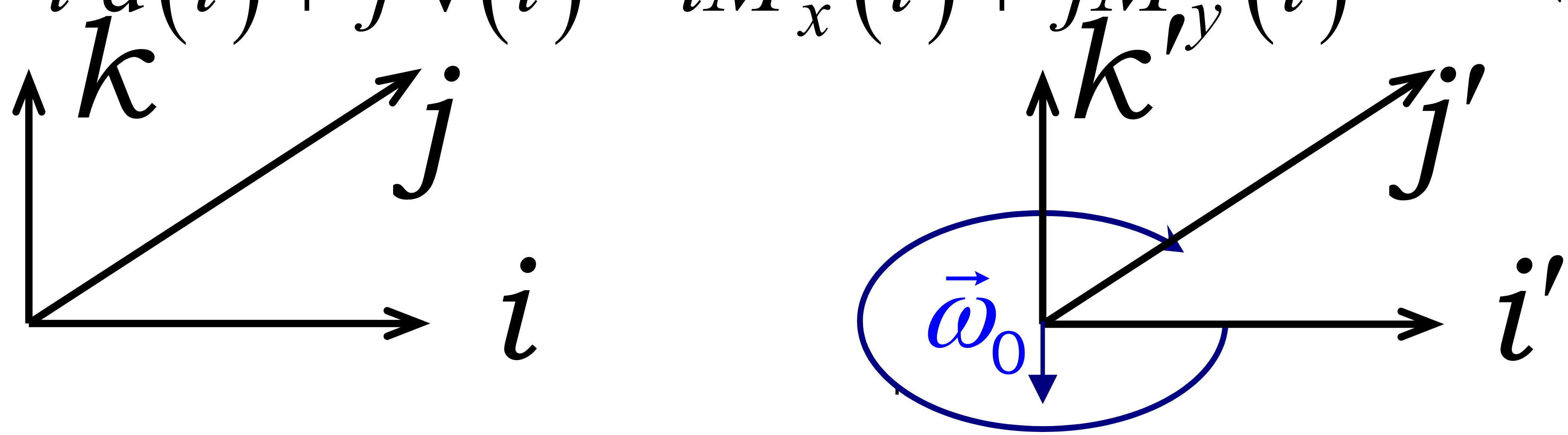
Система координат.

$$\frac{d\vec{M}}{dt} = \gamma\left[\vec{M} \times \vec{B}\right] \qquad \vec{B} = \left\{B_x, B_y, B_0\right\}$$

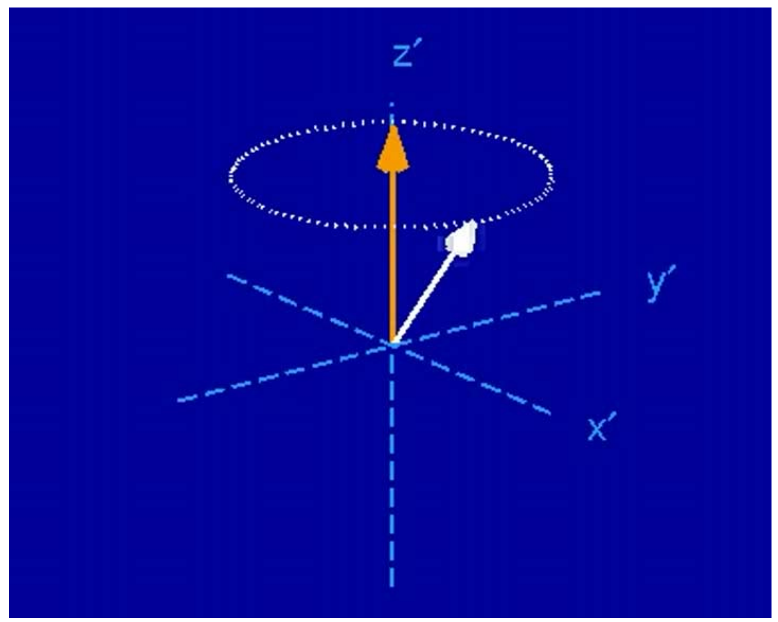
$$\begin{cases} \vec{i}' = \vec{i}\cos(\omega_0 t) - \vec{j}\sin(\omega_0 t) \\ \vec{j}' = -\vec{i}\sin(\omega_0 t) + \vec{j}\cos(\omega_0 t) \\ \vec{k}' = \vec{k} \end{cases}$$

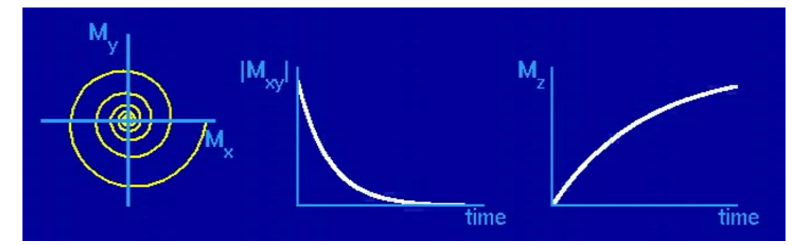
$$\omega = \omega_0$$

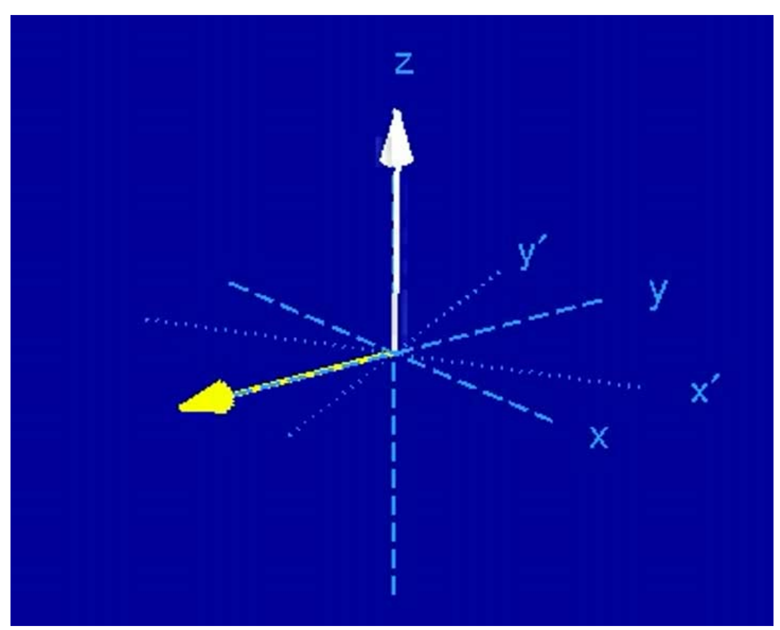
$$\vec{i}'\mathrm{u}(t) + \vec{j}'\mathrm{v}(t) = \vec{i}M_x(t) + \vec{j}M_y(t)$$

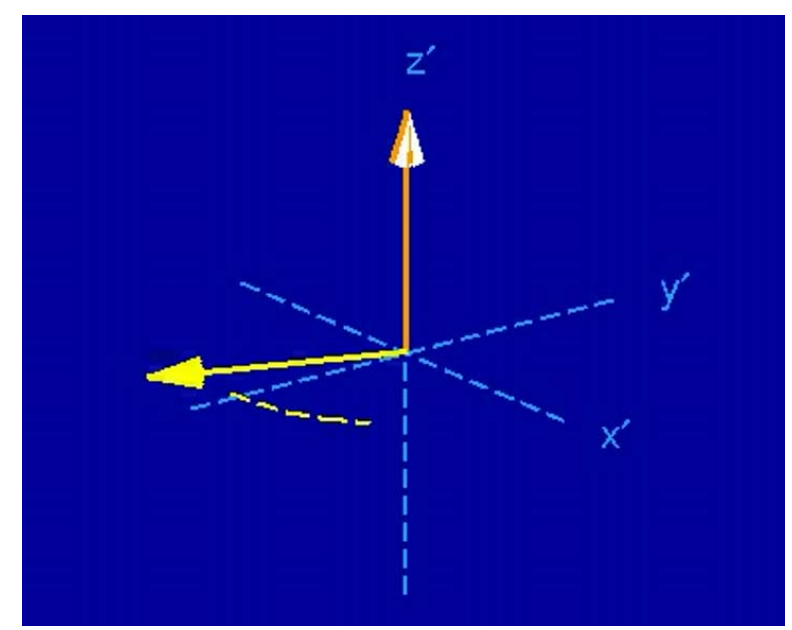


Демо.







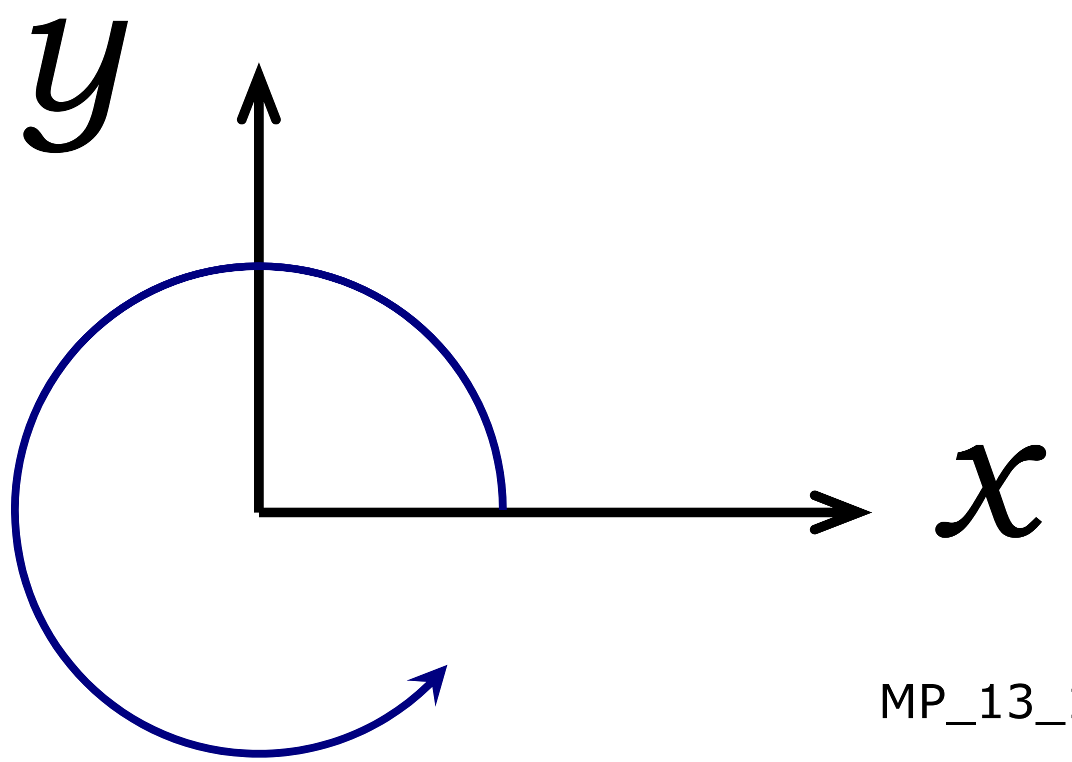


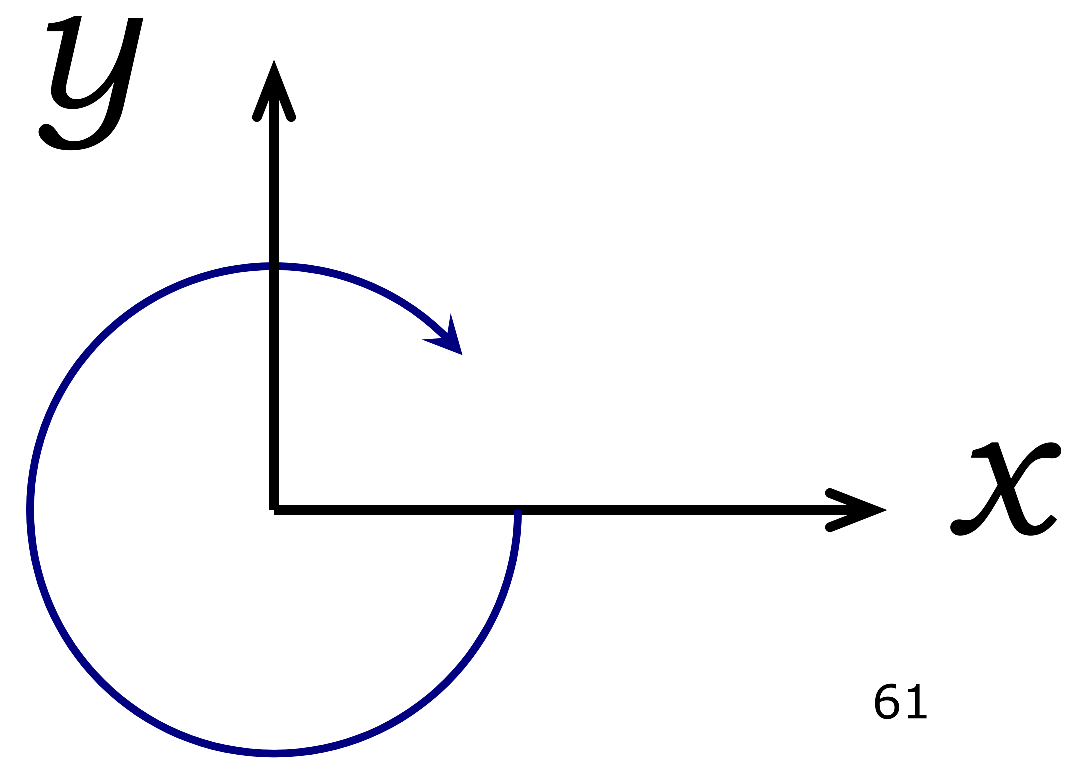
РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\cos(\omega t)\vec{i}$$

$$\vec{B}_1(t) = B_{10}(t)\left[\cos(\omega t)\vec{i} + \sin(\omega t)\vec{j}\right] +$$

$$+B_{10}(t)\left[\cos(\omega t)\vec{i} - \sin(\omega t)\vec{j}\right]$$





РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\cos(\omega t)\vec{i}$$

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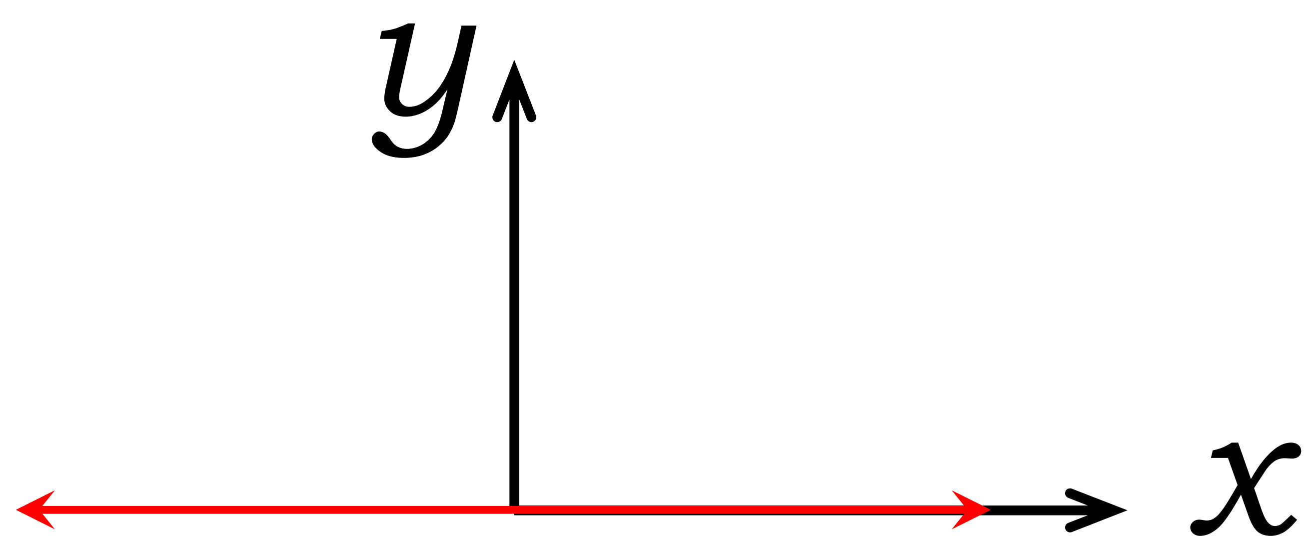
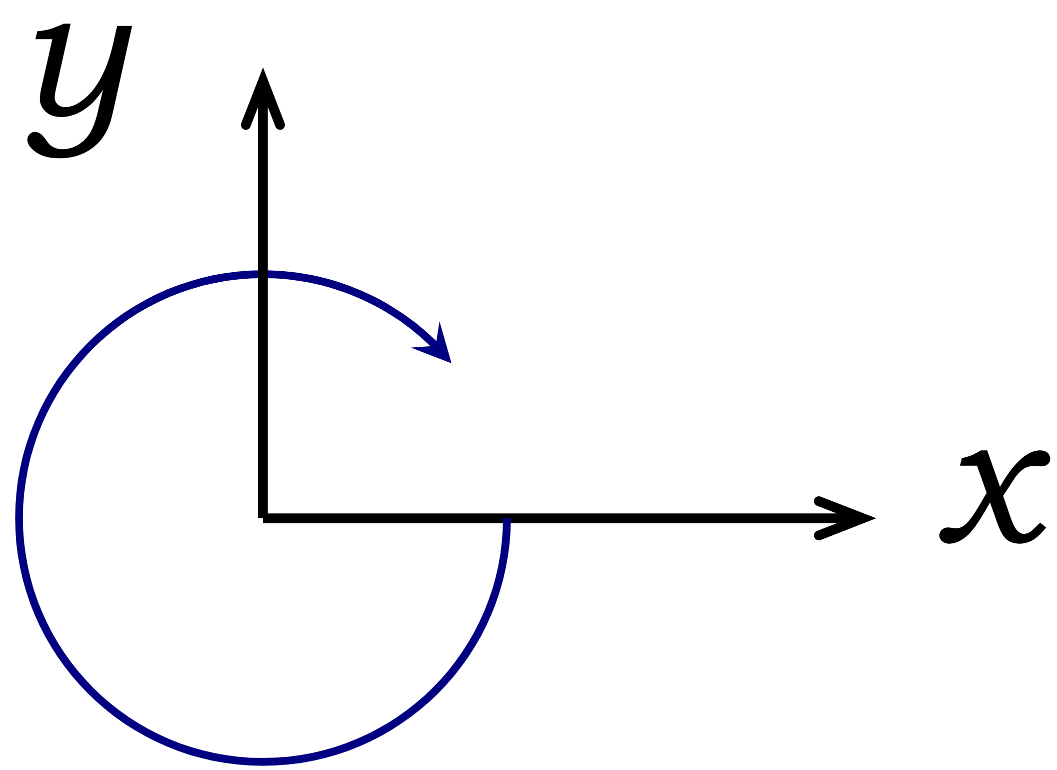
$$+B_{10}(t)\left[\cos(\omega t)\vec{i} - \sin(\omega t)\vec{j}\right]$$



РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\cos(\omega t)\vec{i}$$

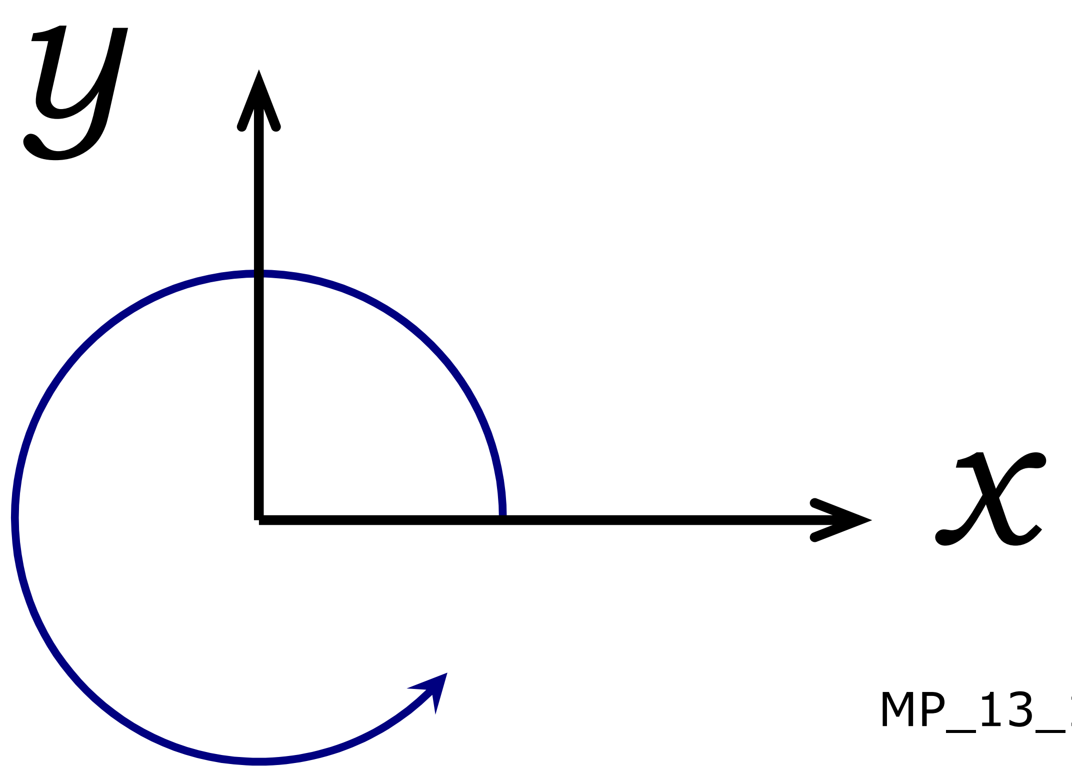
$$\vec{B}_1(t) = B_{10}(t)\left[\cos(\omega t)\vec{i} - \sin(\omega t)\vec{j}\right]$$

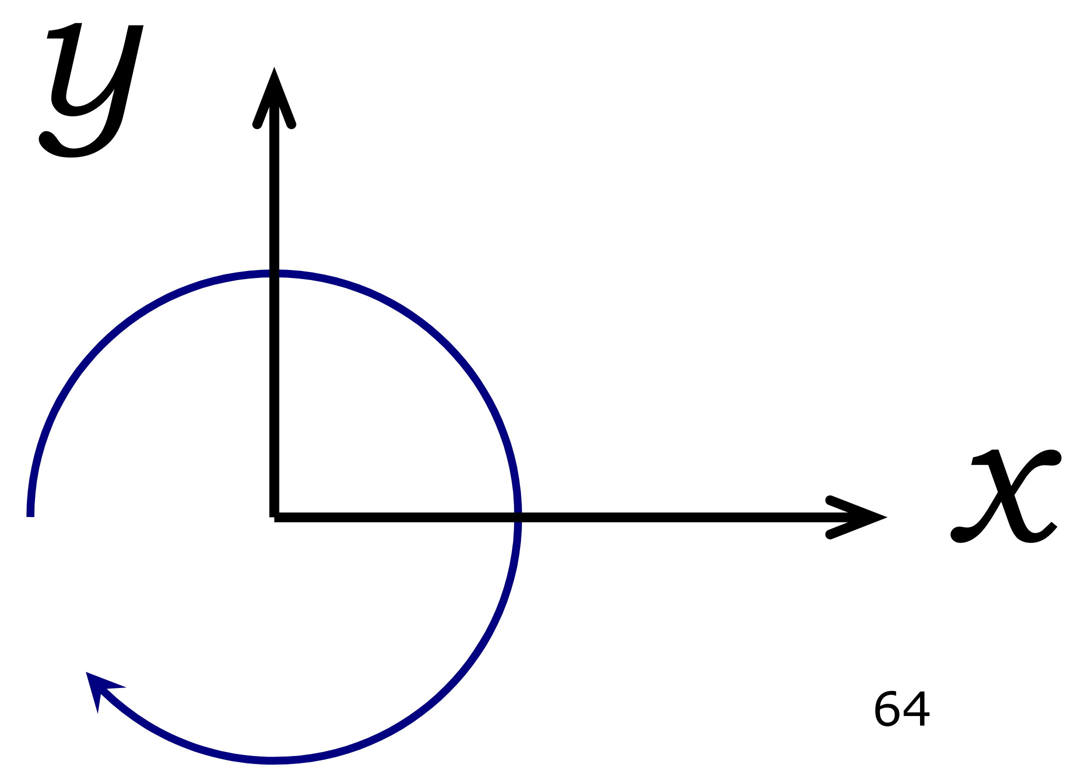


РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\sin(\omega t)\vec{j}$$

$$\vec{B}_1(t) = B_{10}(t)\left[\cos(\omega t)\vec{i} + \sin(\omega t)\vec{j}\right] +$$
$$+B_{10}(t)\left[-\cos(\omega t)\vec{i} + \sin(\omega t)\vec{j}\right]$$

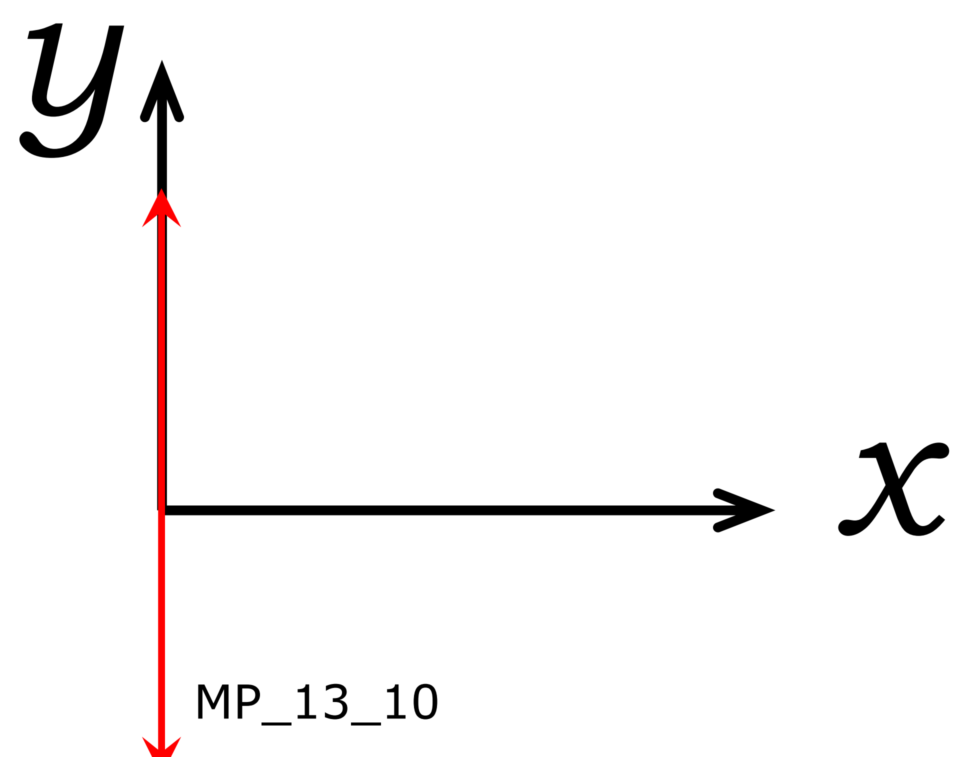




РЧ поле.

$$\vec{B}_1(t) = 2B_{10}(t)\sin(\omega t)\vec{j}$$

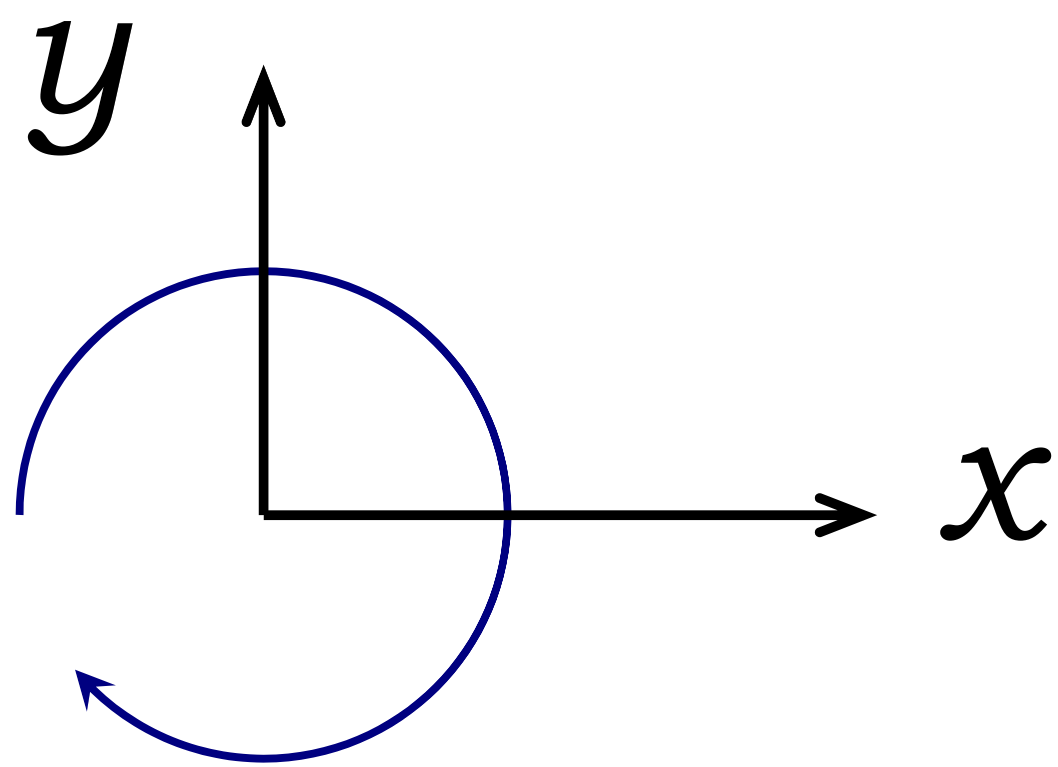
$$\vec{B}_1(t) = B_{10}(t)\left[\cos(\omega t)\vec{i} + \sin(\omega t)\vec{j}\right] +$$
$$+B_{10}(t)\left[-\cos(\omega t)\vec{i} + \sin(\omega t)\vec{j}\right]$$



РЧ поле.

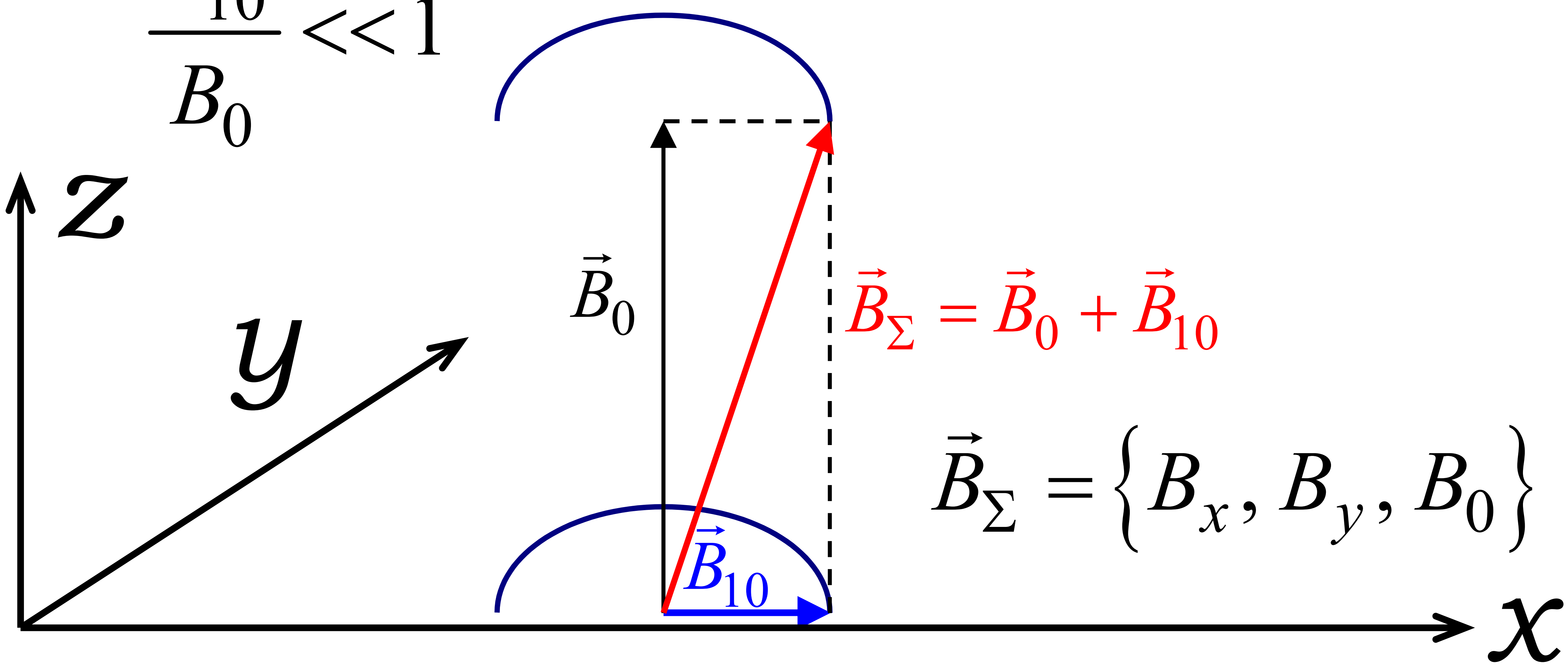
$$\vec{B}_1(t) = 2B_{10}(t)\sin(\omega t)\,\vec{j}$$

$$\vec{B}_1(t) = B_{10}(t)\left[-\cos(\omega t)\,\vec{i} + \sin(\omega t)\,\vec{j}\right]$$



РЧ поле B_1 і стале поле B_0.

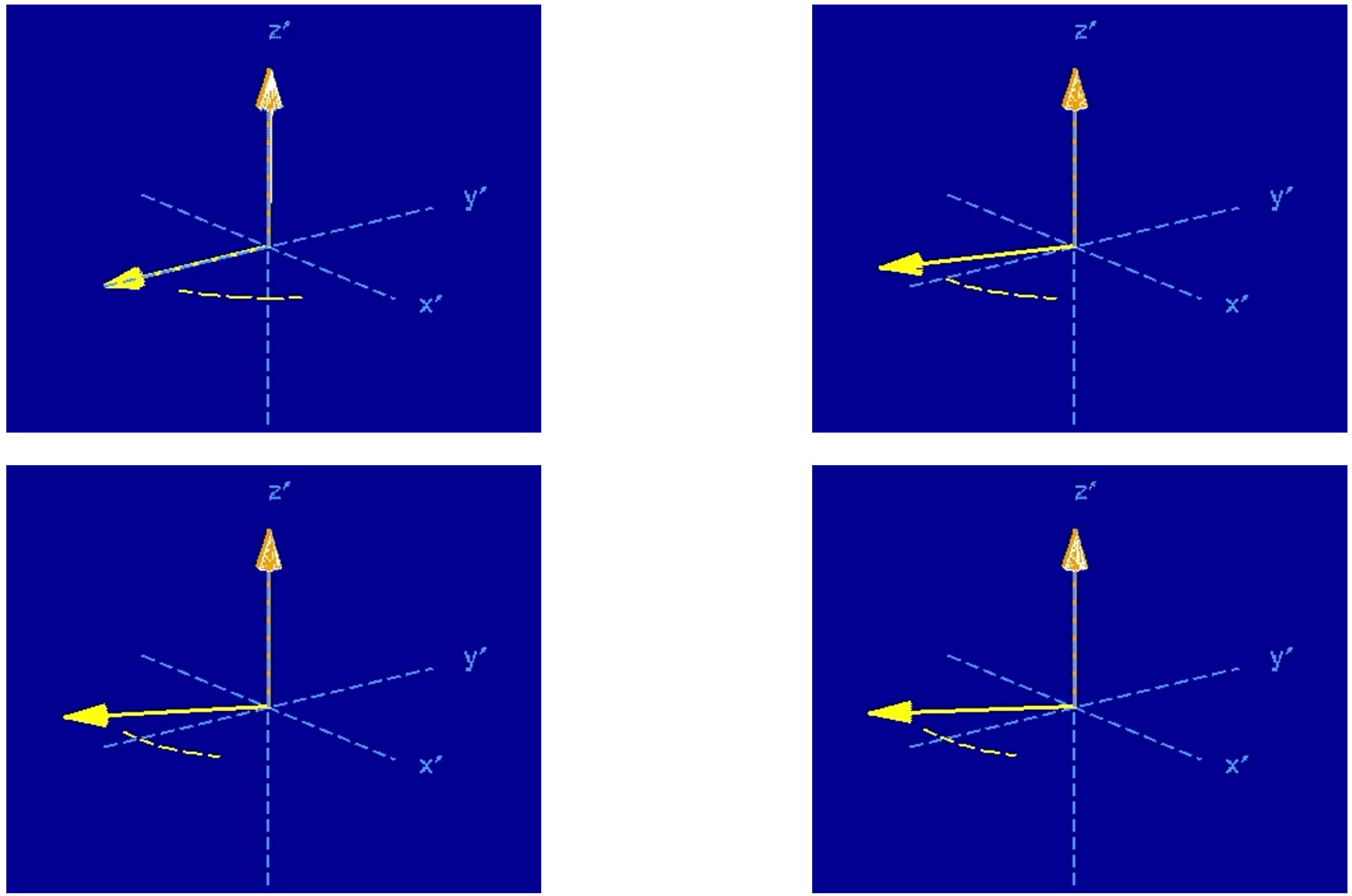
$$\frac{B_{10}}{B_0} << 1$$



$$\vec{B}_\Sigma = \left\{ B_x, B_y, B_0 \right\}$$

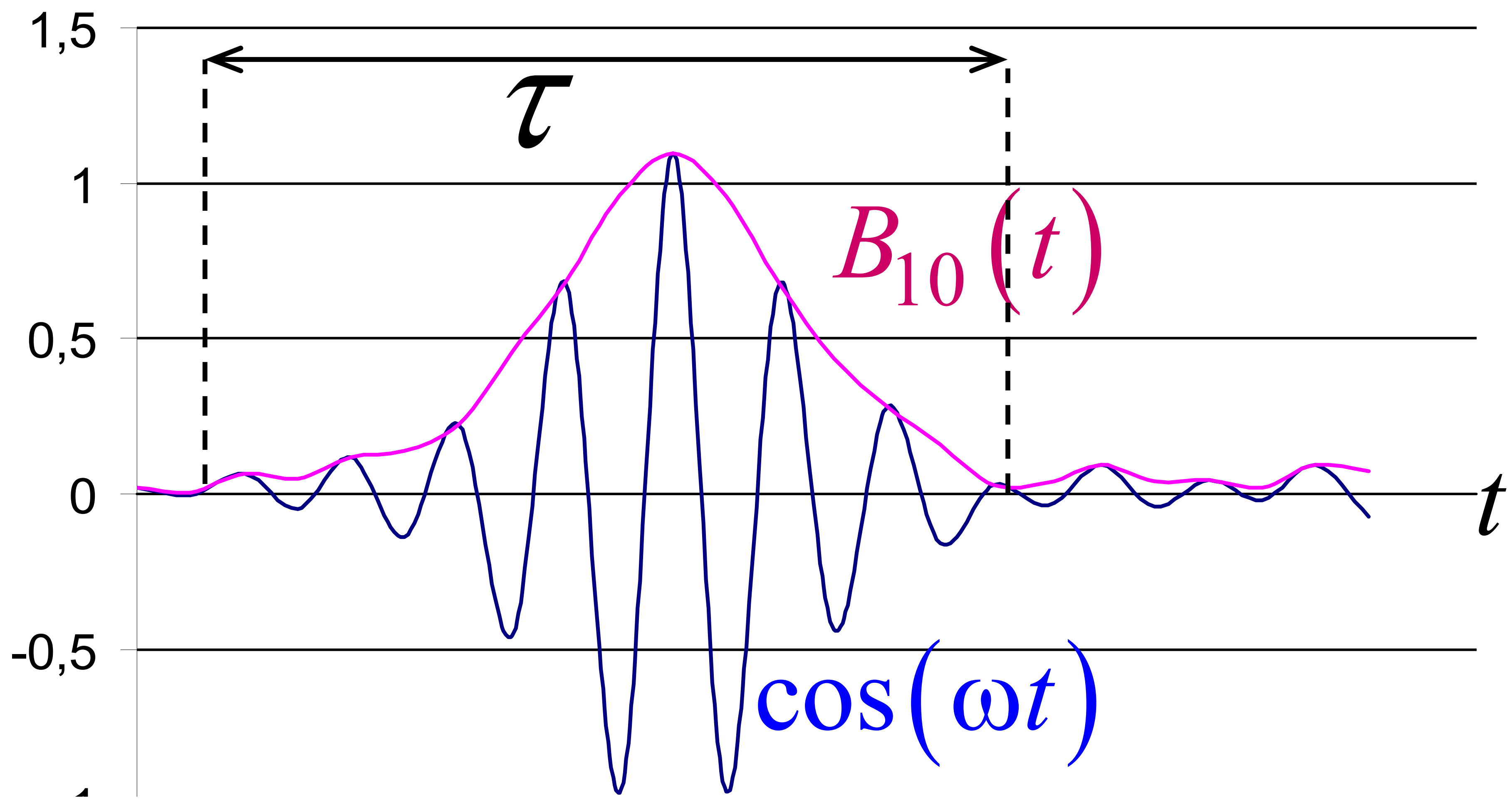
$$\frac{d\vec{M}}{dt} = \gamma \left[\vec{M} \times \vec{B}_\Sigma \right] - \frac{M_x \vec{i} + M_y \vec{j}}{T_2} - \frac{M_z - M_0}{T_1} \vec{k}$$

Демо.



РЧ поле.

$$\vec{B}_1(t) = B_{10}(t)\cos(\omega t)\vec{i}$$

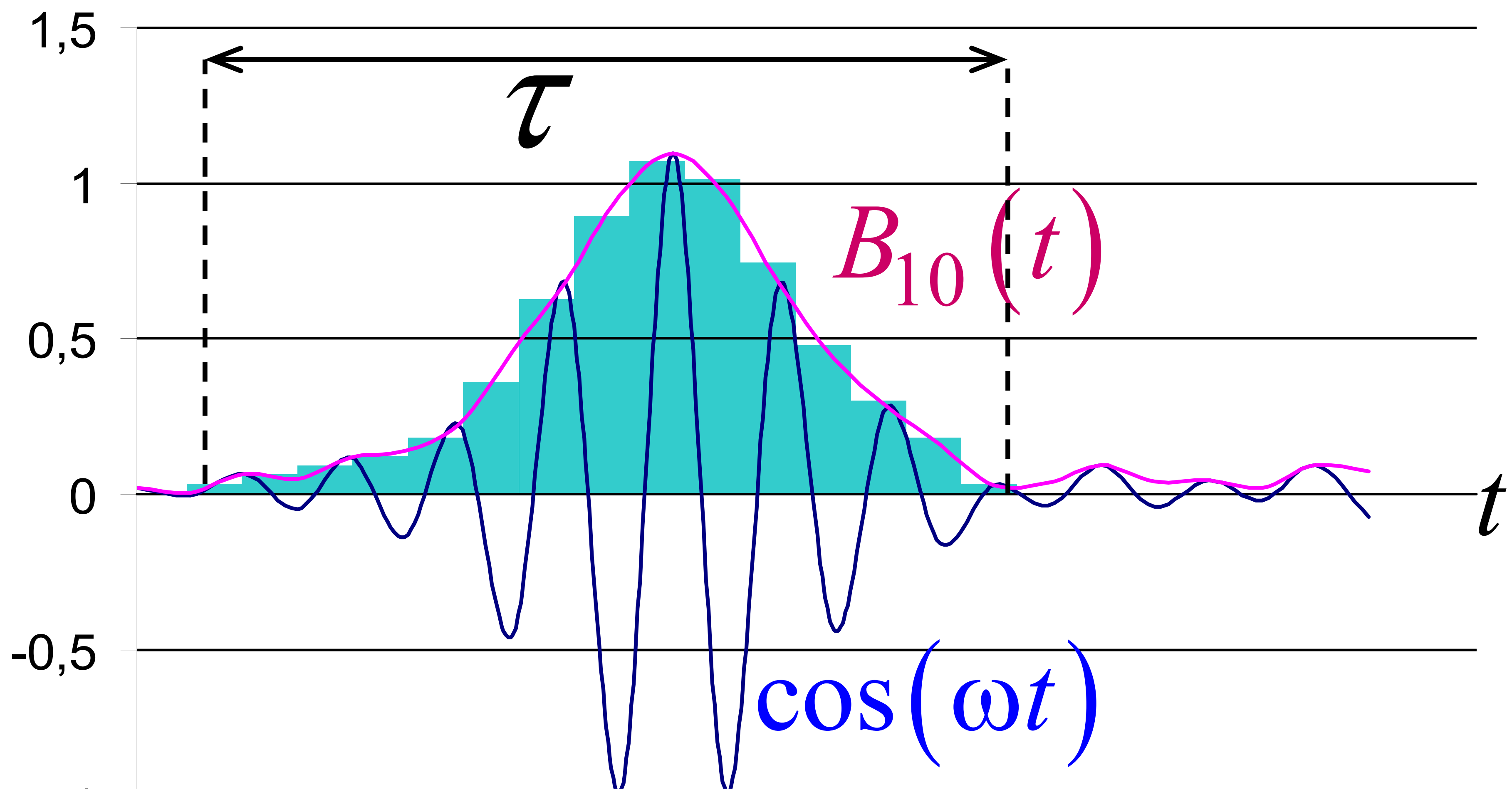


$$\tau > \frac{1}{\gamma B_0}$$

$$B_{10}(t) = \begin{cases} B_{10} = const \neq f(t), 0 \leq t \leq \tau \\ 0, t > \tau \end{cases}$$

РЧ поле.

$$\vec{B}_1(t) = B_{10}(t)\cos(\omega t)\vec{i}$$



$$\tau > \frac{1}{\gamma B_0}$$

$$B_{10}(t) = \begin{cases} B_{10}^{(i)} = const \neq f(t), \tau^{(i-1)} \leq t \leq \tau^{(i)} \\ 0, t > \tau \end{cases}$$

РЧ поле.

$$\vec{B}_1(t) = B_{10}(t)\cos(\omega t)\vec{i}$$

$$\tau > \frac{1}{\gamma B_0}$$

$$\alpha^{(i)} = \tau^{(i)}\gamma B_{10}{}^{(i)}$$

$$\alpha = \sum_{i=1}^{n}\alpha^{(i)} = \sum_{i=1}^{n}\tau^{(i)}\gamma B_{10}{}^{(i)} = \gamma\int_0^{\tau} B_{10}(t)\,dt$$

$$B_{10}(t) = \begin{cases} B_{10}{}^{(i)} = const \neq f(t),\ \tau^{(i-1)} \leq t \leq \tau^{(i)} \\ 0,\ t > \tau \end{cases}$$

МР експеримент.

$$\vec{B} = \left\{ B_x, B_y, B_z \right\} \qquad \vec{B} = \vec{B}_0 + \vec{B}_G + \vec{B}_1$$

$$\vec{B}_0 = \left\{ 0, 0, B_0 \right\}$$

$$\vec{B}_G = \left\{ 0, 0, \vec{G}\vec{r} \right\}$$

$$\vec{G} = \left\{ G_x, G_y, G_z \right\} \quad \vec{r} = \left\{ x, y, z \right\}$$

$$\vec{B}_G = \left(G_x x + G_y y + G_z z \right) \vec{k}$$

$$\vec{B}_1(t) = B_{10}(t) \cos(\omega t) \vec{i}$$

МР експеримент.

$$c(t) = \mathrm{u}(t) + i\mathrm{v}(t) \qquad \vec{B} = \vec{B}_0 + \vec{B}_G + \vec{B}_1$$

$$\frac{dc(t)}{dt} = i\left(\gamma B_G(t)c + \gamma B_1(t)M_0\right)$$

$$\omega_G = -\gamma\left(G_x x + G_y y + G_z z\right)$$

$$c(t) = i\gamma M_0 \exp\left(i\omega_G t\right)\int_0^t \exp\left(-i\omega_G q\right)B_1(q)\,dq$$

Вибір шару.

$$\vec{G} = \{0, 0, G_z\}$$

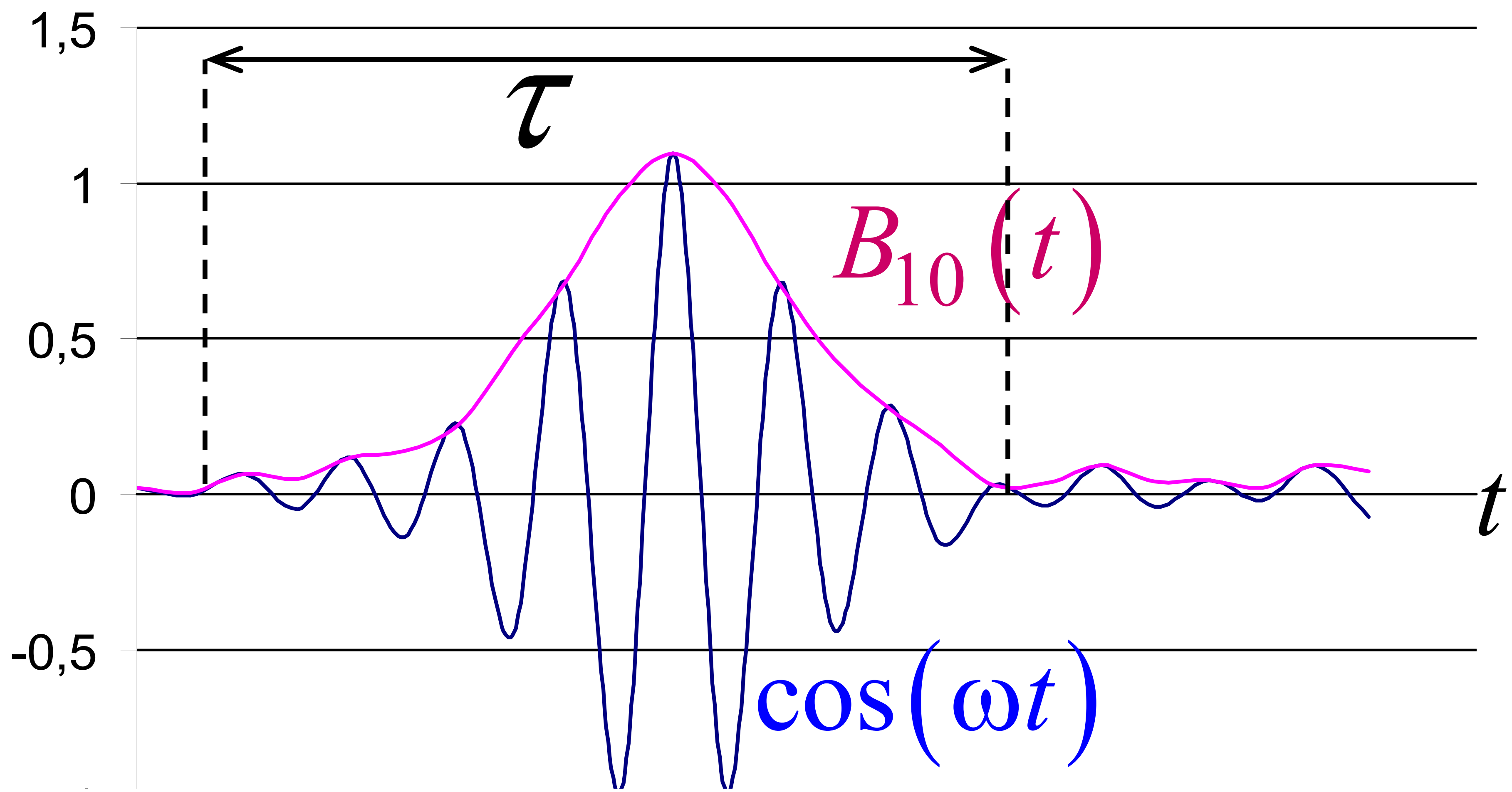
$$\omega_G = -\gamma G_z z$$

$$c(t, z) = i\gamma M_0 \exp(-i\gamma G_z z t) \int_0^t \exp(i\gamma G_z z q) B_1(q) dq$$

$$|c(t, z)| = \begin{cases} A, |z| \le z_0 \\ a, |z| > z_0 \end{cases} \qquad \frac{a}{A} << 1$$

РЧ поле.

$$\vec{B}_1(t) = B_{10}(t)\cos(\omega t)\vec{i}$$



$$\tau > \frac{1}{\gamma B_0}$$

$$B_{10}(t) = \begin{cases} B_{10}{}^{(i)} = const \neq f(t), \tau^{(i-1)} \leq t \leq \tau^{(i)} \\ 0, t > \tau \end{cases}$$

Вибір шару.

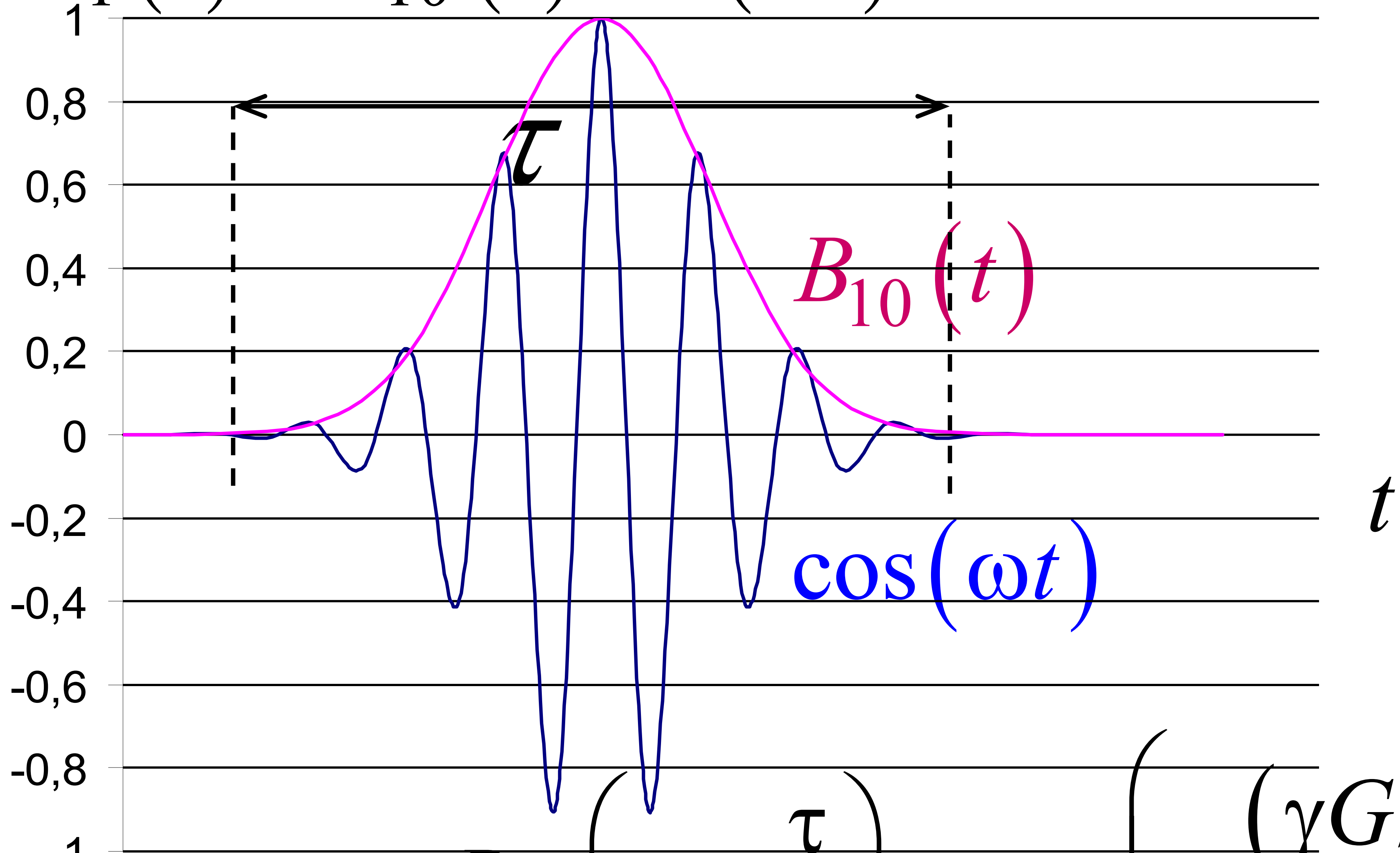
$$c(t,z) = i\gamma M_0 \exp(-i\gamma G_z z t)\int_0^{\tau} \exp(i\gamma G_z z q) B_1(q)\,dq$$

$$|c(t,z)| = \gamma M_0 \left| \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \exp(i\gamma G_z z q) B_1\left(q+\frac{\tau}{2}\right) dq \right|$$

$$|c(t,z)| = \gamma M_0 \left| \int_{-\infty}^{\infty} \exp(i\gamma G_z z q) B_1\left(q+\frac{\tau}{2}\right) dq \right|$$

РЧ поле.

$$\vec{B}_1(t) = B_{10}(t)\cos(\omega t)\vec{i}$$



$$\tau > \frac{1}{\gamma B_0}$$

$$B_{10}\left(q + \frac{\tau}{2}\right) = \exp\left(-\frac{(\gamma G_z dq)^2}{8}\right)$$

Вибір шару.

$$\left|c(t,z)\right| = \gamma M_0 \left| \int_{-\infty}^{\infty} \exp\left(i\gamma G_z z q\right) \exp\left(-\frac{\left(\gamma G_z d q\right)^2}{8} \right) dq \right|$$

$$\left|c(t,z)\right| = \gamma M_0 \frac{G_z}{G_z} \left| \int_{-\infty}^{\infty} \exp\left(i\gamma G_z z q\right) B_1\left(q + \frac{\tau}{2} \right) dq \right|$$

$$\left|c(t,z)\right| = \frac{M_0}{G_z} \left| \int_{-\infty}^{\infty} \exp\left(izQ\right) \exp\left(-\frac{d^2 Q^2}{8} \right) dQ \right|$$

$$Q = \gamma G_z q$$

Вибір шару.

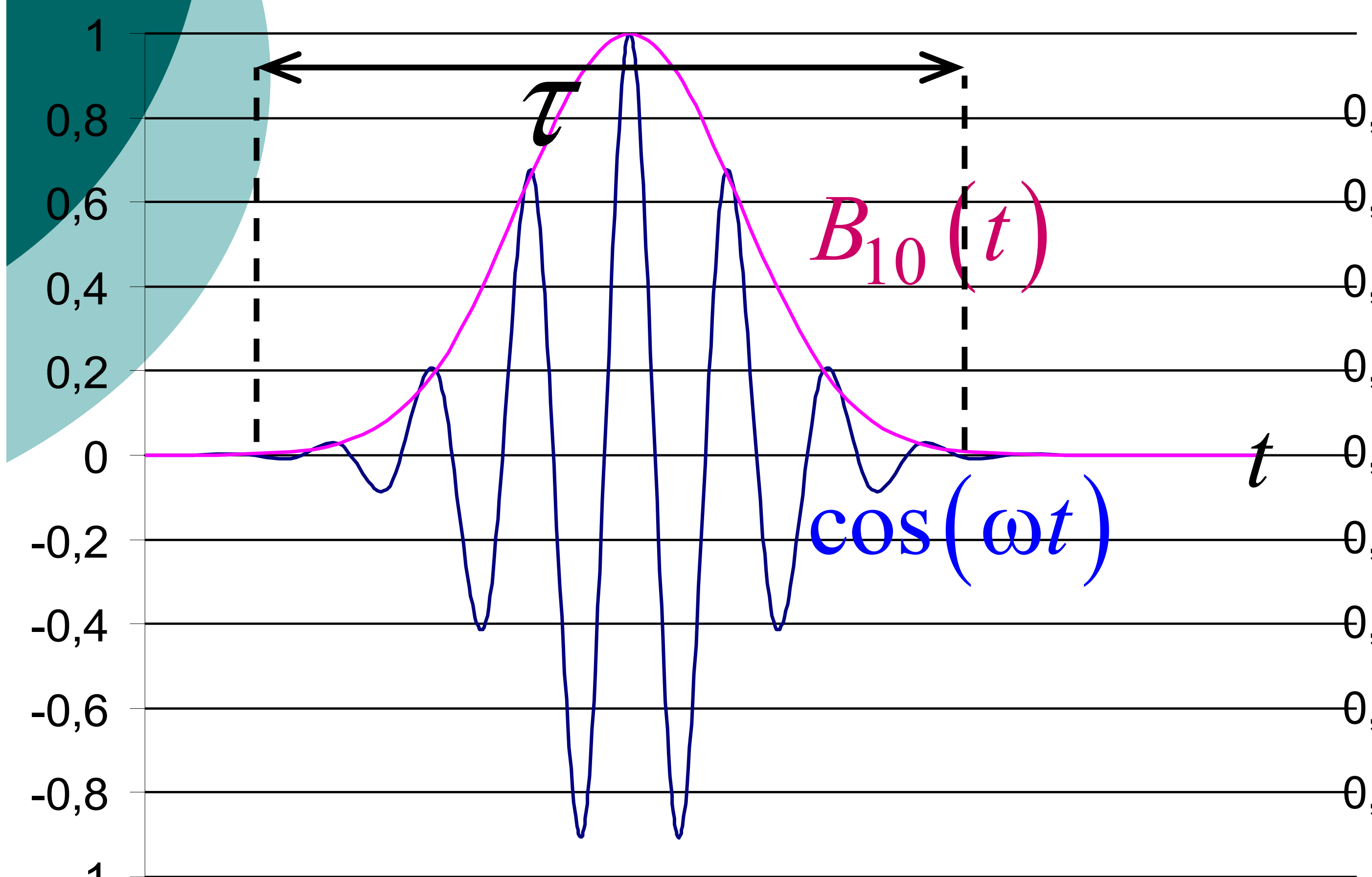
$$\left|c(t,z)\right| = \frac{M_0}{G_z}\left|F\left[\exp\left(-\frac{d^2Q^2}{8}\right)\right]\right|$$

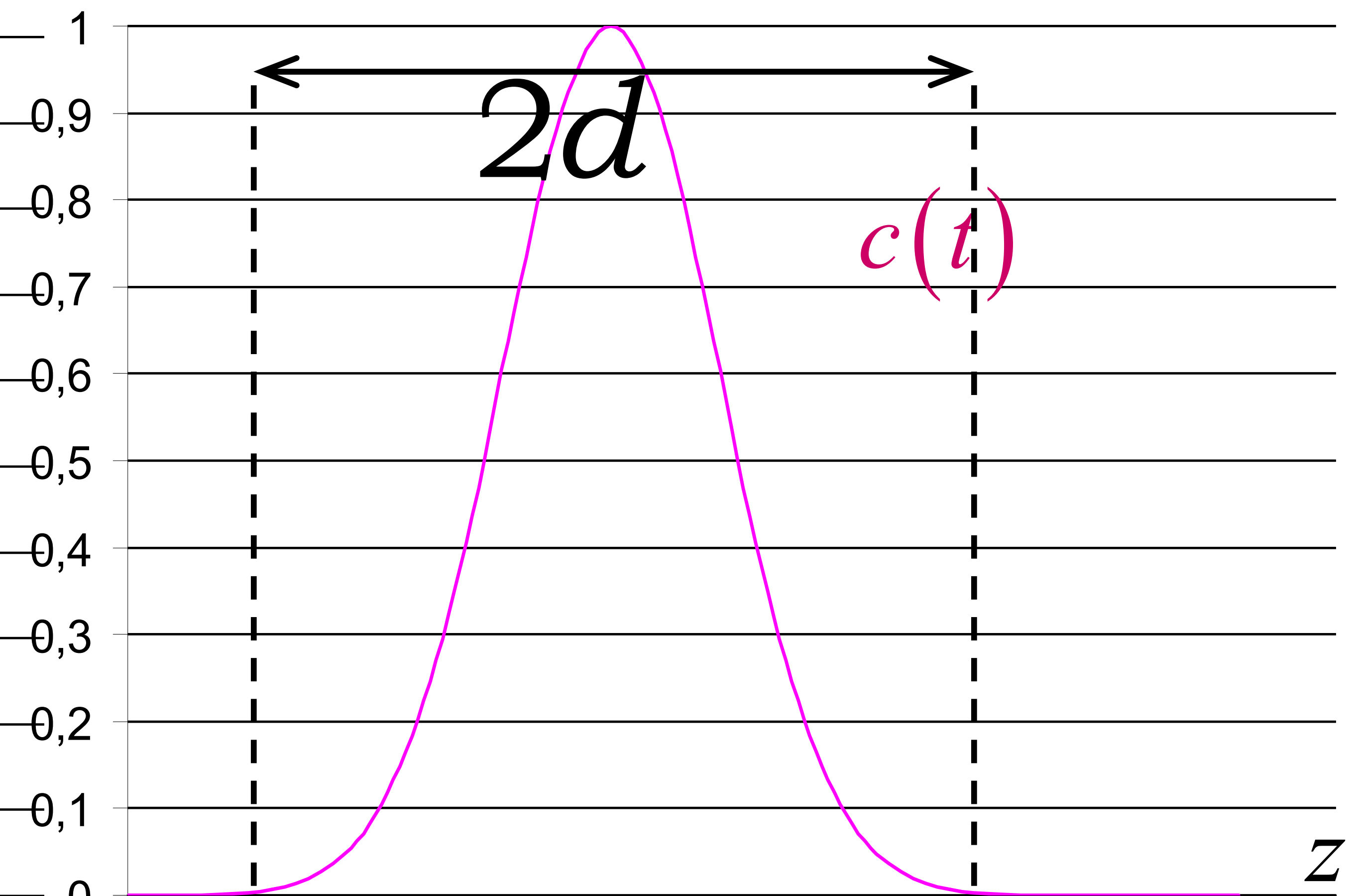
$$F\left[\exp\left(-bt^2\right)\right] = \frac{1}{b}\exp\left(-\frac{\omega^2}{4b}\right) \qquad b = \frac{d^2}{8}$$

$$\left|c(t,z)\right| = \frac{8M_0}{d^2G_z}\left|\exp\left(-\frac{2z^2}{d^2}\right)\right| = \frac{8M_0}{d^2G_z}\exp\left(-\frac{2z^2}{d^2}\right)$$

$$Q = \gamma G_z q$$

РЧ поле і вибір шару.





$$B_{10}\left(q+\frac{\tau}{2}\right)=\exp\left(-\frac{(\gamma G_z d q)^2}{8}\right)$$

$$|c(t,z)|=\frac{8M_0}{d^2 G_z}\exp\left(-\frac{2z^2}{d^2}\right)$$

$z_0 = d$

$$|c(t,z)|=\begin{cases}A, & |z|\le d\\ a, & |z|>d\end{cases}$$

$$\frac{a}{A}<<1$$

Довільний вибір шару.

$$\vec{G} = \{0, 0, G_z\} \Rightarrow \vec{G} = \{G_x, G_y, G_z\}$$

$$\omega_G = -\gamma G_z z \Rightarrow \omega_G = -\gamma\left(G_x x + G_y y + G_z z\right)$$

$$c(t, z) = i\gamma M_0 \exp(-i\gamma G_z z t)\int_0^t \exp(i\gamma G_z z q) B_1(q)\, dq$$

$$|c(t, z)| = \begin{cases} A, & |z| \le z_0 \\ a, & |z| > z_0 \end{cases} \qquad \frac{a}{A} << 1$$

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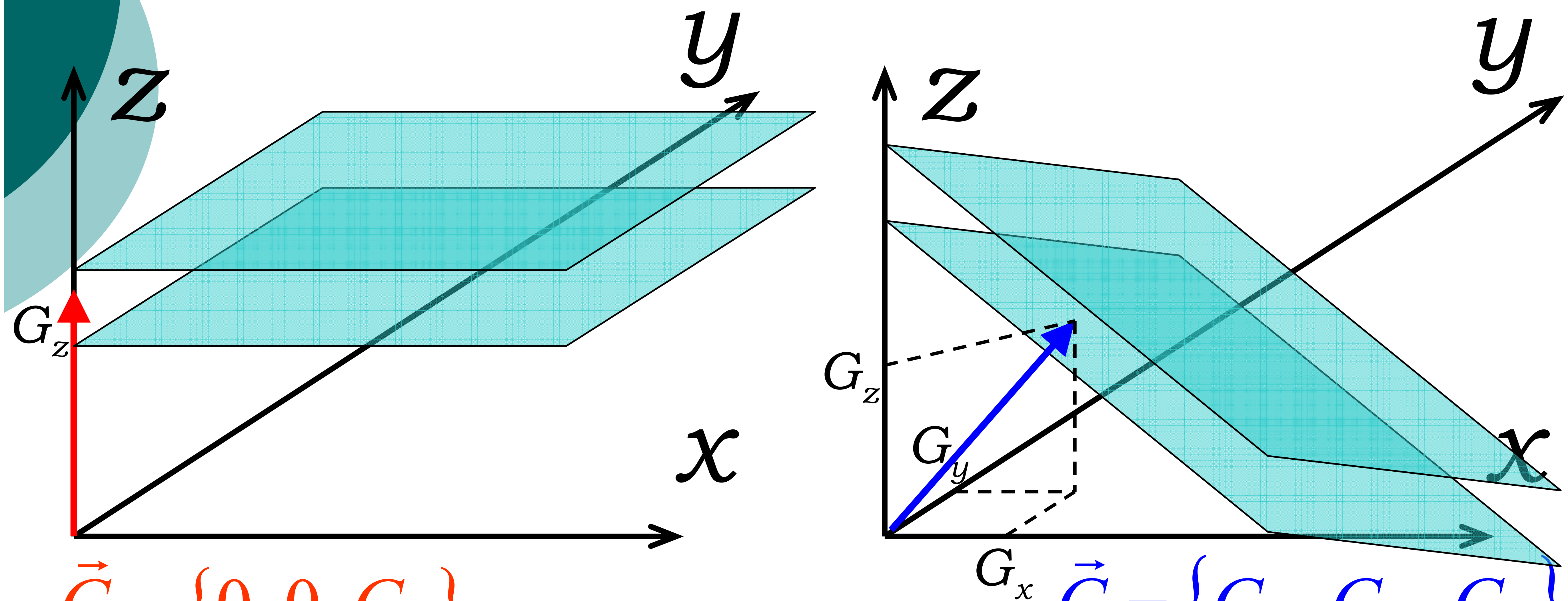
$$c(t, \vec{r}) = i\gamma M_0 \exp\left(-i\gamma \vec{G}\vec{r}t\right)\int_0^t \exp\left(i\gamma \vec{G}\vec{r}q\right)B_1(q)\,dq$$

$$\left|c(t, \vec{r})\right| = \begin{cases} A, \left|\vec{r}\right| \le \vec{r}_0 \\ a, \left|\vec{r}\right| > \vec{r}_0 \end{cases} \qquad \frac{a}{A} << 1$$

Довільний вибір шару.



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$$B_{10}\left(q + \frac{\tau}{2}\right) = \exp\left(-\frac{(\gamma G_z dq)^2}{8}\right) \Rightarrow |c(t, z)| = \frac{8M_0}{d^2 G_z}\exp\left(-\frac{2z^2}{d^2}\right)$$

Довільний вибір шару.

$$c(t,z) = i\gamma M_0 \exp(-i\gamma G_z z t)\int_0^t \exp(i\gamma G_z z q) B_1(q)\,dq$$

$$|c(t,z)| = \begin{cases} A, & |z| \le z_0 \\ a, & |z| > z_0 \end{cases} \qquad \frac{a}{A} << 1$$

$$G = |\vec{G}| = \sqrt{G_x{}^2 + G_y{}^2 + G_z{}^2} \qquad \vec{G}\vec{r} = G_x x + G_y y + G_z z$$

$$z \Rightarrow \vec{G}\vec{r}/G$$

$$c(t,\vec{r}) = i\gamma M_0 \exp(-i\gamma \vec{G}\vec{r} t)\int_0^t \exp(i\gamma \vec{G}\vec{r} q) B_1(q)\,dq$$

$$|c(t,\vec{r})| = \begin{cases} A, & |\vec{G}\vec{r}| \le \vec{G}\vec{r}_0 \\ a, & |\vec{G}\vec{r}| > \vec{G}\vec{r}_0 \end{cases} \qquad \frac{a}{A} << 1$$