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Relativistic quantum field theory

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The relativistic quantum theory of fields was born some thirty-five years ago through the paternal efforts of Dirac, Heisenberg, Pauli and others. It was a somewhat retarded youngster, however, and first reached adolescence seventeen years later, an event which we are gathered here to celebrate. But it is the subsequent development and more mature phase of the subject that I wish to discuss briefly today.

I shall begin by describin g to you the logical foundations of relativistic quantum field theory. No dry recital of lifeless « axioms » is intended but, rather, an outline of its organic growth and development as the synthesis of quantum mechanics with relativity. Indeed, relativistic quantum mechanics - the union of the complementarity principle of Bohr with the relativity principle of Einstein - *is* quantum field theory. I beg your indulgence for the mode of expression I must often use. Mathematics is the natural language of theoretical physics. It is the irreplaceable instrument for the penetration of realms ofphysical phenomena far beyond the ordinary experience upon which conventional language is based.

Improvements in the formal presentation of quantum mechanical principles, utilizing the concept of action, have been interesting by-products of work in quantum field theory. Both my efforts in this direction[1] and those of Feynman[2] (which began earlier) were based on a study of Dirac concerning the correspondence between the quantum transformation function and the classical action. We followed quite different paths, however, and two distinct formulations of quantum mechanics emerged which can be distinguished as differential and integral viewpoints.

In order to suggest the conceptual advantages of these formulations, I shall indicate how the differential version transcends the correspondence principle and incorporates, on the same footing, two different kinds of quantum dynamical variable. It is just these two types that are demanded empirically by the two known varieties of particle statistics. The familiar properties of the variables $q_k, p_k, k = 1 \cdots n$, of the conventional quantumsystem enable one to derive the form of the quantum action principle. It is a differential statement about time transformation functions,

$$\delta\langle t_1|t_2\rangle = (i/\hbar)\langle t_1|\delta[\int_{t_2}^{t_1} \mathrm{d}tL]|t_2\rangle \qquad (1)$$

which is valid for a certain class of kinematical and dynamical variations. The quantum Lagrangian operator of this system can be given the very symmetrical form

$$L = \sum_{k=1}^{n} \frac{1}{4}\left(p_k \frac{\mathrm{d}q_k}{\mathrm{d}t} - q_k \frac{\mathrm{d}p_k}{\mathrm{d}t} + \frac{\mathrm{d}q_k}{\mathrm{d}t} p_k - \frac{\mathrm{d}p_k}{\mathrm{d}t} q_k\right) - H(q, p, t) \qquad (2)$$

The symmetry is emphasized by collecting all the variables into the $2n$ - component Hermitian vector $z(t)$ and writing

$$L = \frac{1}{4}\left(za \frac{\mathrm{d}z}{\mathrm{d}t} - \frac{\mathrm{d}z}{\mathrm{d}t} az\right) - H(z,t) \qquad (3)$$

where a is a real antisymmetrical matrix, which only connects the complementary pairs of variables.

The transformation function depends explicitly upon the choice of terminal states and implicitly upon the dynamical nature of the system. If the latter is held fixed, any alteration of the transformation function must refer to changes in the states, as given by

$$\delta\langle t_1| = (i/\hbar)\langle t_1|G_1 \qquad \delta|t_2\rangle = -(i/\hbar)\, G_2|t_2\rangle \qquad (4)$$

where G_1 and G_2 are infinitesimal Hermitian operators constructed from dynamical variables of the system at the specified times. For a given dynamical system, then,

$$\delta[\int_{t_2}^{t_1} \mathrm{d}tL] = G_1 - G_2 \qquad (5)$$

which is the quantum principle of stationary action, or Hamilton's principle, since there is no reference on the right hand side to variations at intermediate times. The stationary action principle implies equations of motion for the dynamical variables and supplies explicit expressions for the infinitesimal operators $G_{1,2}$. The interpretation of these operators as generators of transformations on states, and on the dynamical variables, implies commutation relations. In this way, all quantum-dynamical aspects of the system are derived from a single dynamical principle. The specific form of the commutation relations obtained from the symmetrical treatment of the usual quantum system is given by the matrix statement

$$[z(t), z(t)] = i\hbar a^{-1} \qquad (6)$$

Note particularly how the antisymmetry of the commutator matches the antisymmetry of the matrix a.

We may now ask whether this general form of Lagrangian operator,

$$L = \frac{1}{4}\left(xA\frac{dx}{dt} - \frac{dx}{dt}Ax\right) - H(x, t) \qquad (7)$$

also describes other kinds of quantum systems, if the properties of the matrix A and of the Hermitian variables x are not initially assigned. There is one general restriction on the matrix A, however. It must be skew - Hermitian, as in the realization by the real, antisymmetrical matrix a. Only one other simple possibility then appears, that of an imaginary, symmetrical matrix. We write that kind of matrix as $i\alpha$, where α is real and symmetrical, and designate the corresponding variables collectively by $\zeta(t)$. The replacement of the antisymmetrical a by the symmetrical α requires that the antisymmetrical commutators which characterize $z(t)$ be replaced by symmetrical anticommutators for $\zeta(t)$, and indeed

$$\{\zeta(t), \zeta(t)\} = \hbar\,\alpha^{-1} \qquad (8)$$

specifies the quantum nature of this second class of quantum variable. It has no classical analogue. The consistency of various aspects of the formalism requires only that the Lagrangian operator be an even function of this second type of quantum variable.

Time appears in quantum mechanics as a continuous parameter which represents an abstraction of the dynamical role of the measurement apparatus. The requirement of relativistic invariance invites the extension of this abstraction to include space and time coordinates. The implication that space-time localized measurements are a useful, if practically unrealizable idealization may be incorrect, but it is a grave error to dismiss the concept on the basis of *a priori* notions of measurability. Microscopic measurement has no meaning apart from a theory, and the idealized measurement concepts that are implicit in a particular theory must be accepted or rejected in accordance with the final success or failure of that theory to fulfill its avowed aims. Quantum field theory has failed no significant test, nor can any decisive confrontation be anticipated in the near future.

Classical mechanics is a determinate theory. Knowledge of the state at a given time permits precise prediction of the result of measuring any property of the system. In contrast, quantum mechanics is only statistically determinate. It is the probability of attaining a particular result on measuring any property of the system, not the outcome of an individual microscopic observation, that is predictable from knowledge of the state. But both theories are causal - a knowledge of the state at one time implies knowledge of the state at a later time. A quantum state is specified by particular values of an optimum set of compatible physical properties, which are in number related to the number of degrees of freedom of the system. In a relativistic theory, the concepts of « before » and « after » have no intrinsic meaning for regions that are in space-like relation. This implies that measurements individually associated with different regions in space-like relation are causally independent, or compatible. Such measurements can be combined in the complete specification of a state. But since there is no limit to the number of disjoint spatial regions that can be considered, a relativistic quantum system has an infinite number of degrees of freedom.

The latter statement, incidentally, contains an implicit appeal to a general property that the mathematics of physical theories must possess - the mathematical description of nature is not sensitive to modifications in physically irrelevant details. An infinite total spatial volume is an idealization of the finite volume defined by the macroscopic measurement apparatus. Arbitrarily small volume elements are idealizations of cells with linear dimensions far below the level of some least distance that is physically significant. Thus, it would be more accurate, conceptually, to assert that a relativistic quantum system has a number of degrees of freedom that is extravagantly large, but finite.

The distinctive features of relativistic quantum mechanics flow from the idea that each small element of three-dimensional space at a given time is physically independent of all other such volume elements. Let us label the various degrees of freedom explicitly - by a point of three-dimensional space (in a limiting sense) ,and by other quantities of finite multiplicity. The dynamical variables then appear as

$$\chi_{a,\mathbf{x}}(t) \equiv \chi_a(t = x^0, \mathbf{x}) \tag{9}$$

which are a finite number of Hermitian operator functions of space-time coordinates, or quantum fields. The dynamical independence of the individual volume elements is expressed by a corresponding additivity of the Lagrangian operator

$$L = \int (d\mathbf{x})\, \mathscr{L} \tag{10}$$

where the Lagrange function $\mathscr{L}$ describes the dynamical situation in the infinitesimal neighborhood of a point. The characteristic time derivative or kinematical part of L appears analogously in $\mathscr{L}$ in terms of the variables associated with the specified spatial point. The relativistic structure of the action principle is completed by demanding that it present the same form, independently of the particular partitioning of space-time into space and time. This is facilitated by the appearance of the action operator, the time integral of the Lagrangian, as the space-time integral of the Lagrange function. Accordingly, we require, as a sufficient condition, that the latter be a scalar function of its field variables, which implies that the known form of the time derivative term is supplemented by similar space derivative contributions. This is conveyed by

$$\mathscr{L} = \frac{1}{4}\left(\chi A^{\mu}\partial_{\mu}\chi - \partial_{\mu}\chi A^{\mu}\chi\right) - \mathscr{H}(\chi) \tag{11}$$

where the A^{μ} are a set of four finite skew-Hermitian matrices. A specific physical field is associated with submatrices of the A^{μ}, which are real and anti - symmetrical for a field j that obeys Bose-Einstein statistics, or imaginary and symmetrical for a field ψ obeying Fermi-Dirac statistics. Finally, the boundaries of the four-dimensional integration region, formed by three-dimensional space at the terminal times, are described by the invariant concept of the space-like surface σ, a three-dimensional manifold such that every pair of points is in space-like relation. The ensuing invariant form of the action principle of relativistic quantum field theory is (we now use atomic units, in which $\hbar = c = 1$)

$$\delta\langle\sigma_1|\sigma_2\rangle = i\langle\sigma_1|\delta\left[\int_{\sigma_2}^{\sigma_1}(\mathrm{d}x)\,\mathscr{L}\right]|\sigma_2\rangle \tag{12}$$

Relativity is a statement of equivalence within a class of descriptions associated with similar but different measurement apparatus. Space-time coordinates are an abstraction of the role that the measurement apparatus plays in defining a space-time frame of reference. The empirical fact, that all connected space-time locations and orientations of the measurement apparatus supply equivalent descriptions, is interpreted by the mathematical requirement ofinvariance under the group of proper orthochronous inhomogenous Lorentz transformations, applied to the continuous numerical coordinates. There is another numerical element in the quantum- mechanical description that has a measure of arbitrariness and expresses an aspect of relativity. I am referring to the quantum-mechanical use of complex numbers and of the mathematical equivalence of the two square roots of - 1, ± i. What general property of any measurement apparatus is subject to our control, in principle, but offers only the choice of two alternatives? The answer is clear - a macroscopic material system can be constructed of matter, or of antimatter! But let us not conclude too hastily that a matter apparatus and an antimatter apparatus are completely equivalent. It is characteristic of quantum mechanics that the dividing line between apparatus and system under investigation can be drawn somewhat arbitrarily, as long as the measurement apparatus always possesses the classical aspects required for the unambiguous recording of an observation. To preserve this feature, the interchange of matter and antimatter must be made on the whole assemblage of macroscopic apparatus and microscopic system. Since the observational label of this duality is the algebraic sign of electric charge, the microscopic interchange must reverse the vector of electric current j^{μ}, while maintaining the tensor $T^{\mu\nu}$ that gives the flux of energy and momentum. But this is just the effect of the coordinate transformation that reflects all four coordinates.

It is indeed true that the action principle does not retain its general form under either of the two transformations, the replacement of i with $-i$, and the reflection of all coordinates, but does preserve it under their combined influence. In more detail, the effect of complex conjugation is equivalent to the reversal of operator multiplication, which distinguishes fields with the two types of statistics. The reflection of all coordinates, a proper transformation, can be generated by rotations in the attached Euclidean space obtained by introducing the imaginary time coordinate $x_4 = i\,x^0$. This transformation alters reality properties, distinguishing fields with integral and half-integral spin. The combination of the two transformations replaces the original Lagrange function

$$\mathscr{L}(\varphi_{\text{int}}, \varphi_{1/2\,\text{int}}, \psi_{\text{int}}, \psi_{1/2\,\text{int}}) \tag{13}$$

with

$$\mathscr{L}(\varphi_{\text{int}}, i\varphi_{1/2\,\text{int}}, i\psi_{\text{int}}, \psi_{1/2\,\text{int}}) \tag{14}$$

If only fields of the types $\varphi_{\text{int}}, \Psi_{\frac{1}{2}\,\text{int}}$ are considered, which is the empirical connection between spin and statistics, the action principle is unaltered in form. This invariance property of the action principles expresses the relativity of matter and antimatter. That is the content of the so-called TCP theorem. The anomalous response of the field types $\varphi_{\frac{1}{2}\,\text{int}}, \Psi_{\text{int}}$ is also the basis for the theoretical rejection of these possibilities as contrary to general physical requirements of positiveness, namely, the positiveness of probability, and the positiveness of energy.

The concept of space-like surface is not limited to plane surfaces. According to the action principle, an infinitesimal deformation of the space-like surface on which a state is specified changes that state by

$$\delta\langle\sigma| = i\langle\sigma|\int d\sigma_\mu T^{\mu\nu}\delta x^\nu \tag{15}$$

which is the infinitely multiple relativistic generalization of the Schrödinger equation

$$\delta\langle t| = i\langle t|H(-\delta t) \tag{16}$$

This set of differential equations must obey integrability conditions, which are commutator statements about the elements of the tensor $T^{\mu\nu}$. Since rigid displacements and rotations can be produced from arbitrary local deformations, the operator expressions of the group properties of Lorentz transformations must be a consequence of these commutator conditions. Foremost among the latter are the equal- time commutators of the energy density T^{00}, which suffice to convey all aspects of relativistic invariance that are not of a three- dimensional nature. A system that is invariant under three-dimensional translations and rotations will be Lorentz invariant if, at equal times[4],

$$-i[T^{00}(x), T^{00}(x')] = -(T^{0k}(x) + T^{0k}(x'))\,\partial_k\,\delta(\mathbf{x}-\mathbf{x}') \tag{17}$$

This is a sufficient condition. Additional terms with higher derivatives of the delta function will occur, in general. But there is a distinguished class of physical system, which I shall call local, for which no further term appears. The phrase « local system » can be given a physical definition within the framework we have used or, alternatively, by viewing the commutator condition as a measurability statement about the property involved in the response of a system to a weak external gravitational field[5]. Only the external gravitational potential g_{00} is relevant here. A physical system is local if the operators $T^{\mu\nu}$, which may be explicit functions of g_{00} at the same time, do not depend upon time derivatives of g_{00}. The class of local systems is limited[6] to fields of spin 0, $\frac{1}{2}$, 1. Such fields are distinguished by their physical simplicity in comparison with fields of higher spin. One may even question whether consistent relativistic quantum field theories can be constructed for non-local systems.

The energy density commutator condition is a very useful test of relativistic invariance. Only a month or so ago I employed it to examine whether a relativistic quantum field theory could be devised to describe magnetic as well as electric charge. Dirac pointed out many years ago that the existence of magnetic charge would imply a quantization of electric charge, in the sense that the product of two elementary charges, $eg/\hbar c$, could assume only certain values. According to Dirac, these values are any integer or half-integer. In recent years, the theoretical possibility of magnetic charge has been attacked from several directions. The most serious accusation is that the concept is in violation of Lorentz invariance. This is sometimes expressed in the language of field theory by the remark that no manifestly scalar Lagrange function can be constructed for a system composed of electromagnetic field and electric and magnetic charge-bearing fields. Now it is true that there is no relativistically invariant theory for arbitary e and g, so that no formally invariant version could exist. Indeed, the unnecessary assumption that $\mathscr{L}$ is a scalar must be relinquished in favor of the more general possibilities that are compatible with the action principle. But the energy commutator condition can still be applied. I have been able to show that energy and momentum density operators can be exhibited which satisfy the commutator condition, together with the three-dimensional requirements, provided $eg/\hbar c$ possesses one of a discrete set of values. These values are integers, which is more restrictive than Dirac's quantization condition. Such general considerations shed no light on the empirical elusiveness of magnetic charge. They only emphasize that this novel theoretical possibility should not be dismissed lightly.

The physical systems that obey the commutator statement oflocality do not include the gravitational field. But, this field, like the electromagnetic field, requires very special consideration. And these considerations make full use of the relativistic field concept. The dynamics of the electromagnetic field is characterized by invariance under gauge transformations, in which the phase of every charge-bearing field is altered arbitrarily, but continuously, at each space-time point while electromagnetic potentials are transformed inhomogeneously. The introduction of the gravitational field involves, not only the use of general coordinates and coordinate transformations, but the establishment at each point of an independent Lorentz frame. The gravitational field gauge transformations are produced by the arbitrary reorientation of these local coordinate systems at each point while gravitational potentials are transformed linearly and inhomogeneously. The formal extension of the action principle to include the gravitational field can be carried out[7], together with the verification of consistency conditions analogous to the energy density commutator conditiona. To appreciate this tour de force, one must realize that the operator in the role of energy density is a function of the gravitational field, which is influenced by the energy density. Thus the object to be tested is only known implicitly. It also appears that the detailed specification of the spatial distribution of energy lacks physical significance when gravitational phenomena are important. Only integral quantities or equivalent asymptotic field properties are physically meaningful in that circumstance. It is in the further study of such boundary conditions that one may hope to comprehend the significance of the gravitational field as the physical mediator between the worlds of the microscopic and the macroscopic, the atom and the galaxy.

I have now spoken at some length about fields. But it is in the language of particles that observational material is presented. How are these concepts related? Let us turn for a moment to the early history of our subject. The quantized field appears initially as a device for describing arbitrary numbers of indistinguishable particles. It was defined as the creator or annihilator of a particle at the specified point of space and time. This picture changed somewhat as a consequence of the developments in quantum electrodynamics to which Feynman, Tomonaga, myself, and many others contributed. It began to be appreciated that the observed properties ofso-called elementary particles are partly determined by the effect of interactions. The fields used in the dynamical description were then associated with noninteracting or bare particles, but there was still a direct correspondence with physical particles. The weakness of electromagnetic interactions, as measured by the small value of the fine structure constant $e^2/\hbar c$ is relevant here, for the same view-point failed disastrously when extended to strongly interacting nucleons and mesons. The resulting wide spread disillusionment with quantum field theory is an unhappy chapter in the history of high-energy theoretical physics, although it did serve to direct attention toward various useful phenomenological calculation techniques.

The great qualitative difference between weakly interacting and strongly interacting systems was impressed upon me by a particular consideration which I shall now sketch for you[9]. In the absence of interactions there is an immediate connection between the quantized Maxwell field and a physical particle of zero mass, the photon. The null mass of the photon is the particle transcription of a field property, that electromagnetism has no well- defined range but weakens geometrically. Now one of the most important interaction aspects of quantum electrodynamics is the phenomenon of vacuum polarization. A variable electromagnetic field induces secondary currents, even in the absence of actual particle creation. In particular, a localized charge creates a counter charge in its vicinity, which partially neutralizes the effect of the given charge at large distances. The implication that physical charges are weaker than bare charges by a universal factor is the basis for charge renormalization. But once the idea of a partial neutralization of charge is admitted one cannot exclude the possibility of total charge neutralization. This will occur if the interaction exceeds a certain strength such that an oppositely charged particle combination, of the same nature as the photon, becomes so tightly bound that the corresponding mass diminishes to zero. Under these circumstances no long-range fields would remain and the massless particle does not exist. We learn that the connection between the Maxwell field and the photon is not an *a priori* one, but involves a specific dynamical aspect, that electromagnetic interactions are weaker than the critical strength. It is a natural speculation that another such field exists which couples more strongly than the critical amount to nucleonic charge, the property carried by all heavy fermions. That hypothesis would explain the absolute stability of the proton, in analogy with the electromagnetic explanation of electron stability, without challenging the uniqueness of the photon.

A field operator is a localized excitation which, applied to the vacuum state, generates all possible energy-momentum, or equivalently, mass states that share the other distinguishing properties of the field. The products of field operators widen and ultimately exhaust the various classes of mass states. If an isolated mass value occurs in a particular product, the state is that of a stable particle with corresponding characteristics. Should a small neighborhood of a particular mass be emphasized, the situation is that of an unstable particle, with a proper lifetime which varies inversely as the mass width of the excitation. The quantitative properties of the stable and unstable particles that may be implied by a given dynamical field theory cannot be predicted with presently available calculation techniques. In these matters, to borrow a phrase of Ingmar Bergman, and St.Paul, we see through a glass, darkly. Yet, in the plausible qualitative inference that a substantial number of particles, stable and unstable, will exist for sufficiently strong interactions among a few fields lies the great promise of relativistic quantum field theory.

Experiment reveals an ever growing number and variety of unstable particles, which seem to differ in no essential way from the stable and long-lived particles with which they are grouped in tentative classification schemes. Surely one must hope that this bewildering complexity is the dynamical manifestation of a conceptually simpler substratum, which need not be directly meaningful on the observational level of particles. The relativistic field concept is a specific realization of this general groping toward a new conception ofmatter.

There is empirical evidence in favor of such simplification at a deeper dynamical level. Strongly interacting particles have been rather successfully classified with the aid of a particular internal symmetry group. It is the unitary group SU_3. The dimensionalities of particle multiplets that have been identified thus far are 1, 8, and 10. But the fundamental multiplet of dimensionality 3 is missing. It is difficult to believe in the physical significance of some transformation group without admitting the existence of objects that respond to the transformations of that group. Accordingly, I would describe the observed situation as follows. There are sets of fundamental fields that form triplets[10] with respect to the group U_3. The excitations produced by these fields are very massive and highly unstable. The low lying mass excitations of mesons and baryons are generated by products of the fundamental fields. If these fields are assigned spin $\frac{1}{2}$, as a specific model, it is sufficient to consider certain products of two and three fields to represent the general properties of mesons and baryons, respectively.

The cogency of this picture is emphasized by its role in clarifying a recent development in symmetry classification schemes. That is the provocative but somewhat mysterious suggestion that the internal symmetry group SU_3 be combined with space- time spin transformations to form the larger unitary group SU_6. This idea, with its relativistic generalizations, has had some striking numerical successes but there are severe conceptual problems in reconciling Lorentz invariance with any union of internal and space-time transformations, as long as one insists on immediate particle interpretation. The situation is different if one can refer to the space-time localizability that is the hallmark of the field concept[11] Let us assume that the interactions among the fundamental fields are of such strength that field products at practically coincident points suffice to describe the excitation of the known relatively low lying particles. The resulting quasi-local structures are in some sense fields that are associated with the physical particles. I call these phenomenological fields, as distinguished from the fundamental fields which are the basic dynamical variables of the system. Linear transformations on the fundamental fields can simulate the effect of external probes, which may involve both unitary and spin degrees of freedom. If these external perturbations are sufficiently gentle, the structure of the particles will be maintained and the phenomenological field transformed linearly with indefinite multiplets. It is not implausible that the highly localized interactions among the phenomenological fields will exhibit a corresponding symmetry. Thus, combined spin and unitary transformations appear as a device for characterizing some gross features of the unknown inner field dynamics of physical particles, as it operates in the neighborhood of a specific point. But these transformations can have no general significance for the transfer of excitations from point to point, and only lesser symmetries will survive in the final particle description.

Phenomenological fields are the basic concept in formulating the practical calculation methods of strong interaction field theory. They serve to isolate the formidable problem of the dynamical origin of physical particles from the more immediate questions referring to their properties and interactions. In somewhat analogous circumstances, those of non-relativistic many-particle physics, the methods and viewpoint of quantum field theory[12] have been enormously successful. They have clarified the whole range of cooperative phenomena, while employing relatively simple approximation schemes. I believe that phenomenological relativistic quantum field theory has a similar future, and will replace the algorithms that were introduced during the period of revolt from field theory. But the intuition that serves so well in non-relativistic contexts does not exist for these new conditions. One has still to appreciate the precise rules of phenomenological relativistic field theory, which must supply aself-consistent descriptionof the residual interactions, given that the strong fundamental interactions have operated to compose the various physical particles. And when this is done, how much shall we have learned, and how much will remain unknown, about the mechanism that builds matter from more primitive constituents? Are we not at this moment,

. . . like stout Cortez when with eagle eyes
He star'd at the Pacific - and all his men
Look'd at each other with a wild surmise -
Silent, upon a peak in Darien.

And now it only remains for me to say: Tack så mycket för uppmärksamheten.

1. Some references are: J.Schwinger, *Phys.Rev.,* 82 (1951) 914, 91 (1953) 713; *Proc. Natl. Acad Sci.(U. S.),* 46 (1960) 883.
2. R.Feynman, *Rev. Mod. Phys.,* 20 (1948) 36; R.Feynman and A.Hibbs, *Quantum Mechanics and Path Integrals,* McGraw-Hill, New York, 1965.
3. In the first two papers cited in Ref.1 I have assumed space-reflection invariance and shown the equivalence between the spin-statistics relation and the invariance of the action principle under combined time reflection and complex conjugation. It was later remarked by Pauli that the separate hypothesis of space-reflection invariance

was unnecessary, W.Pauli, *Niels Bohr and the Development of Physics,* McGraw-Hi& New York, 1955.
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11. J. Schwinger, *Second Coral Gables Conference on Symmetry Principles at High Energy,* Freeman, San Francisco, 1965; *Phys. Rev.,* 140 (1965) B 158.
12. The general theory is described by P. Martin and J. Schwinger, *Phys. Rev.,* 115 (1959) 1342.