Білет 3

(4) $Z = q_0 V T^4$

$\frac{C_p}{C_V} - ?$

$F = -kT \ln Z$

$C_p = T\left(\frac{\partial S}{\partial T}\right)_p \qquad C_V = T\left(\frac{\partial S}{\partial T}\right)_V$

$S = -\left(\frac{\partial F}{\partial T}\right)_V$

$F = -kT \ln q_0 V T^4 = -kT[\ln q_0 V + 4\ln T]$

$S = +k[\ln q_0 V + 4\ln T] + kT\frac{4}{T} =$

$= +k \ln q_0 V T^4 + 4k$

P

P

$S = k \ln q_0 V + 4k\ln T + 4k$

$C_V = T\left(\frac{\partial S}{\partial T}\right) = T\frac{4k}{T} = 4k$

$p = nkT = \frac{N}{V}kT \qquad V = \frac{NkT}{p}$

$S = k \ln g_0 V T^4 + 4k =$ Білет 3

$= k \ln g_0 \frac{N k T T^4}{P} + 4k =$

$= k \ln g_0 \frac{N k}{P} + 5k \ln T + 4k =$

$C_p = T \left(\frac{\partial S}{\partial T} \right)_p = T \frac{5k}{T} = 5k$

$\frac{C_p}{C_V} = \frac{5}{4}.$

Білет 19

(4) Знайти $\langle \Delta T^2 \rangle$ — ? для ідеал. газу

$$W \sim \exp\left[-\frac{1}{2}\,\frac{\Delta S \Delta T - \Delta p \Delta V}{kT}\right]$$

$$\Delta S = \left(\frac{\partial S}{\partial T}\right)_V \Delta T + \left(\frac{\partial S}{\partial V}\right)_T \Delta V$$

$$\Delta p = \left(\frac{\partial p}{\partial T}\right)_V \Delta T + \left(\frac{\partial p}{\partial V}\right)_T \cdot \Delta V$$

$$\left(\frac{\partial S}{\partial T}\right)_V T = C_V \,; \quad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$W \sim \exp\left\{-\frac{1}{2kT}\left[C_V \frac{\Delta T^2}{T} + \left(\frac{\partial p}{\partial T}\right)_V \Delta V \Delta T - \right.\right.$$

$$\left.\left. - \Delta V \left(\frac{\partial p}{\partial T}\right)_V \Delta T - \left(\frac{\partial p}{\partial V}\right)_T \Delta V^2\right]\right\} =$$

$$= \exp\left\{-\frac{1}{2}\,\frac{1}{kT}\left[C_V \frac{\Delta T^2}{T} - \left(\frac{\partial p}{\partial V}\right)_T \Delta V^2\right]\right\}$$

$$W \sim \exp\left\{-\frac{1}{2}\sum_{i,j} a_{ij} x_i x_j\right\}$$

$$\langle x_i x_j \rangle = (a^{-1})_{ij}$$

$$\langle x_i x_i \rangle = (a^{-1})_{ii}$$

$$W \sim \exp\left[-\frac{C_V \Delta T^2}{2kT^2} + \frac{\left(\frac{\partial p}{\partial V}\right)_T \cdot \Delta V^2}{kT}\right]$$

$$\hat{a} = \begin{pmatrix} \frac{C_V}{kT} & 0 \\ 0 & -\frac{\left(\frac{\partial p}{\partial V}\right)_T}{kT} \end{pmatrix} \quad a^{-1} = \begin{pmatrix} \frac{kT^2}{C_V} & 0 \\ 0 & -kT\left(\frac{\partial V}{\partial p}\right)_T \end{pmatrix}$$

$$\langle \Delta T^2 \rangle = \frac{kT^2}{C_V}$$

Білет № 5

(3) $\left(\frac{\partial T}{\partial p}\right)_H$ - ? в газі, що задовільняє рівнянню Дітерічі.

$$p(V-b) = RT\exp\left(-\frac{a}{RTV}\right)$$

Білет 19

1) Знайти умови існування одинарної петлі гістерезису для фаз. переходів I роду

Тер. пот. Гіббса

$$G(p_z) = G_0 + E_z p_z + a(T - T_c) p_z^2$$

Умова існування $\frac{\partial G(p_z)}{\partial p_z} = 0$

$$E_z + 2a(T - T_c) p_z = 0$$

$$p_z = \frac{E_z}{2a(T_c - T)}$$

Білет 23

4) Знайти $\langle \Delta V \Delta T \rangle$ в змінних S і p

$$\Delta V = \left(\frac{\partial V}{\partial S}\right)_p \Delta S + \left(\frac{\partial V}{\partial p}\right)_S \Delta p$$

$$\Delta T = \left(\frac{\partial T}{\partial S}\right)_p \Delta S + \left(\frac{\partial T}{\partial p}\right)_S \Delta p$$

$$\Delta p \Delta V - \Delta T \Delta S = \cancel{\left(\frac{\partial V}{\partial p}\right)_S \left(\frac{\partial T}{\partial S}\right)}$$

$$\Delta p \Delta S \left(\frac{\partial V}{\partial S}\right)_p + \left(\frac{\partial V}{\partial p}\right)_S \Delta p^2 - \left(\frac{\partial T}{\partial S}\right)_p \Delta S^2 -$$

$$- \left(\frac{\partial T}{\partial p}\right)_S \Delta p \Delta S =$$

$$= \left(\frac{\partial V}{\partial p}\right)_S \Delta p^2 - \left(\frac{\partial T}{\partial S}\right)_p \Delta S^2 + \Delta p \Delta S \left(\left(\frac{\partial V}{\partial S}\right)_p - \left(\frac{\partial T}{\partial p}\right)_S\right)$$

$$\langle \Delta S^2 \rangle = kT \left(\frac{\partial S}{\partial T}\right)_p \qquad \langle \Delta p^2 \rangle = -kT \left(\frac{\partial p}{\partial V}\right)_S$$

$$\langle \Delta V \Delta T \rangle = \left(\frac{\partial V}{\partial S}\right)_p \left(\frac{\partial T}{\partial S}\right)_p \langle \Delta S \rangle^2 +$$ Білет 19

$$+ \left(\frac{\partial V}{\partial p}\right)_S \left(\frac{\partial T}{\partial p}\right)_S \langle \Delta p \rangle^2$$

$$\langle \Delta S \Delta p \rangle = 0$$

$$\langle \Delta V \Delta T \rangle = kT \left(\frac{\partial V}{\partial S}\right)_p - kT \left(\frac{\partial T}{\partial p}\right)_S$$

Ŧ.5.

(4) $\cancel{Z_N} = \sum_i z_i = \int e^{-\frac{\sum \varkappa_i}{kT}} \prod_i d\Gamma_i = \int \prod_i e^{-\frac{\varkappa_i}{kT}} d\Gamma_i = \prod_i z_i = Z_1^N$

$$Z_1 = \sum_{\{n\}} e^{-\frac{\varepsilon_i}{kT}} = e^{-\frac{E_1}{kT}} + e^{-\frac{E_2}{kT}}$$

$pV = kNT$

$$Z = Z_1^N = \left(e^{-E_1/kT} + e^{-E_2/kT}\right)^N$$

$$\cancel{F = -kT \ln Z = -kTN \ln\left(e^{-E_1/kT} + e^{-E_2/kT}\right)}$$

$$\cancel{C_V = T\left(\frac{\partial S}{\partial T}\right)_V} \;;\; \cancel{S = -\left(\frac{\partial F}{\partial T}\right)_V}$$

$$p_n = \frac{1}{Z} e^{-\varepsilon_n/kT} = \frac{e^{-\varepsilon_n/kT}}{\left(e^{-E_1/kT} + e^{-E_2/kT}\right)}$$

$$\langle E \rangle = \sum_n p_n E_n = \frac{1}{Z} \sum_n e^{-\varepsilon_n/kT} \varepsilon_n =$$

$$= \frac{\varepsilon_1 e^{-\varepsilon_1/kT} + \varepsilon_2 e^{-\varepsilon_2/kT}}{e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}} \;;\; C = \frac{dE}{dT} =$$

$$= \frac{\left(e^{-\varepsilon_1/kT} + \varepsilon_1\left(-\frac{1}{kT}\right) e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT} - \frac{\varepsilon_2}{kT} e^{-\varepsilon_2/kT}\right)\left(e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right) - \left(\varepsilon_1 e^{-\varepsilon_1/kT} + \varepsilon_2 e^{-\varepsilon_2/kT}\right)\left(-\frac{1}{kT} e^{-\varepsilon_1/kT} - \frac{1}{kT} e^{-\varepsilon_2/kT}\right)}{\left(e^{-\varepsilon_1/kT} + e^{-\varepsilon_2/kT}\right)^2} =$$

$$= \frac{\left(1 - \frac{\varepsilon_1}{kT} + e^{-\varepsilon_2/kT + \varepsilon_1/kT} - \frac{\varepsilon_2}{kT} e^{-\varepsilon_2/kT + \varepsilon_1/kT}\right)\left(1 + e^{-\varepsilon_2/kT + \varepsilon_1/kT}\right) - \left(\varepsilon_1 + \varepsilon_2 e^{-\varepsilon_2/kT + \varepsilon_1/kT}\right)\left(-\frac{1}{kT} - \frac{1}{kT} e^{-\varepsilon_2/kT + \varepsilon_1/kT}\right)}{\left(1 + e^{-\varepsilon_2/kT + \varepsilon_1/kT}\right)^2}$$

$e^{\cdots} \equiv t$

$$= \frac{1 \cancel{+t} \cancel{-\frac{\varepsilon_1}{kT}} \cancel{-\frac{\varepsilon_1}{kT} t} \cancel{+t} + t^2 - \frac{\varepsilon_2}{kT} t - \cancel{\frac{\varepsilon_2}{kT} t^2} \cancel{+\varepsilon_1 \frac{1}{kT}} + \cancel{\frac{\varepsilon_1}{kT} t} \cancel{+\frac{\varepsilon_2 t}{kT}} \cancel{+\frac{\varepsilon_2 t^2}{kT}}}{(1+t)^2} =$$

$$= \frac{1+t^2}{(1+t)^2} \;;\; t = e^{-\varepsilon_2/kT + \varepsilon_1/kT}$$

Білет 22

Є осцилятор одновимірний гармонічний, ω_0 — власна частота при T. Знайти F і побудувати графік.

$$E_n = \hbar\omega_0\left(n+\frac{1}{2}\right), \quad n \geq 0$$

$$g_n = 1$$

$$Z = \sum_n g_n e^{-\frac{E_n}{kT}} = \sum_{n=0}^{\infty} e^{-\frac{\hbar\omega_0}{kT}\left(n+\frac{1}{2}\right)}$$

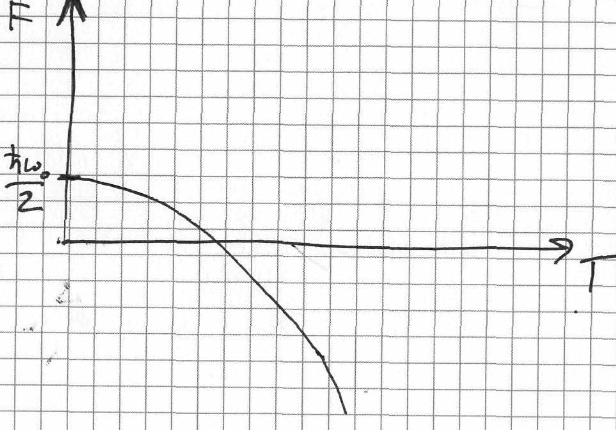
$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

$$Z = \frac{1}{1-e^{-\frac{\hbar\omega_0}{kT}}} e^{-\frac{\hbar\omega_0}{2kT}}$$

$$F = -kT\ln Z$$

$$\ln Z = -\frac{\hbar\omega_0}{2kT} - \ln\left(1-e^{-\frac{\hbar\omega_0}{kT}}\right)$$

$$F = \frac{\hbar\omega_0}{2} + kT\ln\left(1-e^{-\frac{\hbar\omega_0}{kT}}\right).$$



Білет 6

4) $\langle \Delta S \, \Delta p \rangle$ - ? при T, V

$$\Delta S = \left(\frac{\partial p}{\partial V}\right)_T \cdot \Delta V + \left(\frac{\partial p}{\partial T}\right)_V \cdot \Delta T$$

$$\Delta p = \left(\frac{\partial p}{\partial T}\right)_V \Delta V + \left(\frac{\partial p}{\partial V}\right)_T \Delta T$$

$$\Delta p \cdot \Delta S = \left(\left(\frac{\partial p}{\partial T}\right)_V \Delta V + \left(\frac{\partial p}{\partial V}\right)_T \Delta T\right) \times$$

$$\times \left(\left(\frac{\partial p}{\partial V}\right)_T \Delta V + \underbrace{\left(\frac{\partial p}{\partial T}\right)_V \Delta T}_{\frac{C_V}{T}\Delta T}\right) =$$

$$= \left(\frac{\partial p}{\partial V}\right)_T \cdot \left(\frac{\partial p}{\partial T}\right)_V \Delta V^2 + \left(\frac{\partial p}{\partial V}\right)_T \frac{C_V \Delta T \Delta V}{T} +$$

$$+ \left(\frac{\partial p}{\partial T}\right)_V^2 \Delta V \Delta T +$$

$$+ \frac{C_V \Delta T^2}{T} \left(\frac{\partial p}{\partial T}\right)_V$$

$$\langle \Delta T^2 \rangle = \frac{kT^2}{C_V}, \quad \langle \Delta V^2 \rangle = -kT \left(\frac{\partial V}{\partial p}\right)_T$$

$$\langle \Delta T \Delta V \rangle = 0.$$

$$\langle \Delta p \cdot \Delta S \rangle = -kT \left(\frac{\partial V}{\partial p}\right)_V \cdot \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial p}{\partial T}\right)_V +$$

$$+ \frac{C_V}{T} \left(\frac{\partial p}{\partial T}\right)_V \cdot \frac{kT^2}{C_V} = 0.$$

Білет 13

(3) Знайти ентропію осциляторів при T і рез. частотою ω.

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right), \quad n \geq 0$$

$$g_n = 1$$

$$Z = \sum_n g_n e^{-\frac{E_n}{kT}} = \sum_{n=0} e^{-\frac{\hbar\omega}{kT}\left(n+\frac{1}{2}\right)}$$

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

$$Z = \frac{1}{1-e^{-\frac{\hbar\omega}{kT}}} e^{-\frac{\hbar\omega}{2kT}}$$

$$F = -kT \ln Z$$

$$F = \frac{\hbar\omega}{2} + kT \ln\left(1-e^{-\frac{\hbar\omega}{kT}}\right)$$

$$S = -\frac{dF}{dT} = -\left(k\ln\left(1-e^{-\frac{\hbar\omega}{kT}}\right) - \frac{kT\frac{\hbar\omega}{kT^2}}{1-e^{-\frac{\hbar\omega}{kT}}}\right)$$

$$= \frac{\hbar\omega}{T}\frac{1}{1-e^{-\frac{\hbar\omega}{kT}}} - k\ln\left(1-e^{-\frac{\hbar\omega}{kT}}\right).$$

Білет 17

3 $\langle \Delta T \cdot \Delta p \rangle$ для ідеального газу

$\langle \Delta T \cdot \Delta p \rangle$:

$$\Delta p = \left(\frac{\partial p}{\partial V}\right)_T \Delta V + \left(\frac{\partial p}{\partial T}\right)_V \Delta T$$

$$\Delta p \, \Delta T = \left(\frac{\partial p}{\partial V}\right)_T \Delta V \Delta T + \left(\frac{\partial p}{\partial T}\right)_V \Delta T^2$$

$$\langle \Delta p \, \Delta T \rangle = \left(\frac{\partial p}{\partial T}\right)_V \Delta T^2 =$$

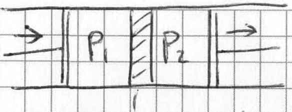
$$\Delta T^2 = \frac{kT^2}{C_V}$$

$$= \left(\frac{\partial p}{\partial T}\right)_V \frac{kT^2}{C_V} =$$

$$p = nkT$$

$$\langle \Delta p \, \Delta T \rangle = \frac{nkkT^2}{C_V} = n\frac{k^2T^2}{C_V}$$

№21



пориста перегородка

$$E_2 - E_1 = P_1V_1 - P_2V_2$$

$$E_1 + P_1V_1 = E_2 + P_2V_2$$

тобто $H_1 = H_2$

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{\partial(T,H)}{\partial(P,H)} = \frac{\frac{\partial(T,H)}{\partial(P,T)}}{\frac{\partial(P,H)}{\partial(P,T)}} = -\left(\frac{\partial H}{\partial P}\right)_T \left(\frac{\partial H}{\partial T}\right)_P^{-1}$$

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

Ефект виникає в неідеальних газах

№ 2

$$P(x) = C(x^2 + bx^4)e^{-\frac{x^2}{a^2}}$$

C − ?

$\langle \Delta x^2 \rangle$ − ?

$$\int\limits_{-\infty}^{+\infty} P(x)\,dx = 1$$ − умова нормування

$$C\int\limits_{-\infty}^{+\infty}(x^2+bx^4)e^{-\frac{x^2}{a^2}}dx = 1$$

$$C\Bigg(\underbrace{\int\limits_{-\infty}^{+\infty} x^2 e^{-\frac{x^2}{a^2}}dx}_{\overset{\shortparallel}{\mathring{I}_1}} + b\underbrace{\int\limits_{-\infty}^{+\infty} x^4 e^{-\frac{x^2}{a^2}}dx}_{\overset{\shortparallel}{\mathring{I}_2}}\Bigg) = 1$$

$$\mathring{I} = \int\limits_{-\infty}^{+\infty} e^{-\alpha x^2}dx = \sqrt{\frac{\pi}{\alpha}}$$

$$\mathring{I}_1 = -\frac{\partial}{\partial\alpha}\mathring{I} = -\left(\sqrt{\pi}\,\frac{-1}{2\alpha\sqrt{\alpha}}\right)\Bigg|^{\alpha=\frac{1}{a^2}} = \frac{\sqrt{\pi}}{2}a^3$$

$$\mathring{I}_2 = -\frac{\partial}{\partial\alpha}\mathring{I}_1 = -\left(\frac{\sqrt{\pi}}{2}\,\frac{-3}{2\alpha^{5/2}}\right) = \frac{3}{4}\sqrt{\pi}\,a^5 \Rightarrow$$

$$C = \frac{1}{\mathring{I}_1 + b\,\mathring{I}_2}$$

$$\langle \Delta x^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle x \rangle = \int\limits_{-\infty}^{+\infty} \underbrace{x}_{\text{не парна}}\underbrace{P(x)}_{\text{парна}}\,dx = 0$$

$$\langle \Delta x^2 \rangle = \langle x^2 \rangle = \int\limits_{-\infty}^{+\infty} x^2\, C\left(x^2 + bx^4\right) e^{-\frac{x^2}{a^2}}\, dx =$$

$$= C \cdot \Big[\underbrace{\int\limits_{-\infty}^{+\infty} x^4 e^{-\frac{x^2}{a^2}} dx}_{= I_2'} + b \underbrace{\int\limits_{-\infty}^{+\infty} x^6 e^{-\frac{x^2}{a^2}} dx}_{= I_3'} \Big]$$

$$I_3' = -\frac{\partial}{\partial \alpha} I_2' = \frac{3}{4}\sqrt{\pi}\, \frac{5}{2\alpha^{7/2}} = \frac{15}{8}\sqrt{\pi}\, a^7$$

$$\langle \Delta x^2 \rangle = C \cdot \left[I_2' + b\, I_3' \right]$$

Вилітають незалежно
Імовірність вильоту 1 e
за час Δt дорівнює b
Знайти P вильоту N e за час t.

$$P_N(t) = \frac{(bt)^N}{N!} e^{-bt}$$ розподіл Пуассона

$$\begin{cases} \dfrac{dP_N}{dt} = \left(P_{N-1} - P_N\right) b \\ \dfrac{dP_0}{dt} = -b P_0 \end{cases}$$

$$P_0(t) = e^{-bt} \Rightarrow \frac{dP_0}{dt} = -b e^{-bt} = -b P_0$$

$$\frac{dP_N}{dt} = \frac{1}{N!}\left(N (bt)^{N-1} b e^{-bt} - b (bt)^N e^{-bt} \right) =$$

$$= b \underbrace{\frac{(bt)^{N-1}}{(N-1)!} e^{-bt}}_{= P_{N-1}} - b \underbrace{\frac{(bt)^N}{N!} e^{-bt}}_{= P_N} =$$

②

① Білет №27. т. п. Гібса

Знайти $G(T,p)$ через стат. суму z.

Розв'язок.

$$E = F + TS \Rightarrow F = E - TS$$

$$\underline{G = F + pV}$$

$$E = G - pV + TS;$$

$$\underline{S = k_0 \ln z + \frac{E}{T};}$$

$$\underbrace{k_0 \ln z} = TS - E = -F;$$

$$F = G - pV$$

$$-G + pV = k_0 \ln z;$$

Виразимо p:

$$dE = dA + \delta Q;$$

$(T = const)$

$$\Delta E = \left\{ \langle \Delta E \rangle = \Delta\left(\sum_m E_m \omega_m\right) \right. = \underbrace{\sum_m E_m \Delta\omega_m}_{\Delta Q = 0} + \underbrace{\sum_m \omega_m \Delta E_m}_{\Delta A}$$

$$p = -\frac{dA}{dV} = \overline{\frac{\Delta A}{\Delta V}} = -\sum_m \omega_m \frac{\Delta E_m}{\Delta V}\bigg|_{T = const} = ;$$

$$= \left[w_m = \frac{1}{z} e^{-\frac{E_m}{k_0 T}} \right] = -\sum_m \frac{1}{z} e^{-\frac{E_m}{k_0 T}} \frac{\Delta E_m}{\Delta V}\Bigg|_{T = const} =$$

$$= \left[\begin{array}{l} \Delta E_m \to 0;\ \Delta V_m \to dV_m \\ \text{тому } \Delta E_m \approx dE_m \\ \Delta V_m \approx dV_m \end{array} \right] =$$

$$= -\sum_m \frac{1}{z} e^{-\frac{E_m}{k_0 T}} \frac{dE_m}{dV_m} = \left[e^{-\frac{E_m}{k_0 T}} dE_m = -k_0 T\, d e^{-\frac{E_m}{k_0 T}} \right] =$$

$$= -\frac{1}{z} \sum_m (-k_0 T) \frac{d e^{-\frac{E_m}{k_0 T}}}{dV_m} = \frac{k_0 T}{z} \frac{d\left(\sum e^{-E_m/k_0 T}\right)}{dV_m} =$$

$$= \frac{k_0 T}{z} \frac{dz}{dV_m} = \underbrace{k_0 T \cdot \frac{d \ln z}{dV}\Bigg|_{T = const}}_{= p}.$$

$$-G + pV = k_0 \ln z;$$

$$-G + k_0 T \left(\frac{\partial \ln z}{\partial V} \right)_T \cdot V = k_0 \ln z;$$

$$G = k_0 \left[T \left(\frac{\partial (\ln z)}{\partial V} \right)_T V - \ln z \right]$$

---

(2) $p(x) - ?$ $\quad x \in (0; +\infty)$.

$S_0 - ?$

$\bar{x} = x_0$; $\quad S_{\bar{x} = x_0} = S_{max}$;

$$\tilde S(p(x)) \quad \tilde S = S_0 - \alpha\Big(\underbrace{\int_0^{+} x\,p(x)\,dx}_{\bar x} - x_0\Big) - \beta\Big(\underbrace{\int_0^{+} p(x)\,dx}_{=1} - 1\Big)$$

$$\underline{S_0 = -k_0\int_0^{+\infty} p(x)\ln p(x)\,dx}$$

$$\frac{\partial \tilde S}{\partial p(x)} = 0;$$

$$0 = \int -k_0\Big(\ln p(x) + \frac{\cancel{p(x)}}{\cancel{p}}\Big) - \alpha x - \beta;$$

$$\Downarrow$$

$$\ln p(x) = \frac{-\alpha x - \beta}{k_0} - 1;$$

$$\boxed{p(x) = C\cdot e^{-\frac{\alpha}{k_0}x}} \qquad \underline{\underline{\alpha,\ C;}}\ ??$$

$$\begin{cases} \underline{\bar x = x_0 = \int_0^{+} x\,p(x)\,dx} \Rightarrow \underline{\underline{C = \frac{1}{x_0}}};\ \underline{\underline{\alpha = \frac{k_0}{x_0}}} \\ \underline{\int_0^{+} p(x)\,dx = 1} \end{cases} \qquad \underline{p(x) = \frac{1}{x_0}e^{-\frac{x}{x_0}}}$$

$$\Downarrow$$

$$S_0 = -k_0\cdot\int_0^{+\infty} p(x)\ln p(x)\,dx =$$

$$= \ldots$$

Б. 2

(2) $p_0(t+\Delta t) = p_0(t)\cdot(1-p_1)$

λ - дуже мала, тому вважаємо, що за час dt ввійде більше одного гост. вірогід. мала (нехт.).

$$P_n(t+dt) = P_{n-1}(t)\cdot P_1 + P_n(t)\cdot(1-P_1)$$

З іншого боку $P_n(t+dt) = P_n(t)\cdot$
$\cdot\, dt + P_n(t) \Rightarrow$

$$\begin{cases} \dfrac{dP_n(t)}{dt} = \left(P_{n-1}(t) - P_n(t)\right)p_1 = \\ = \lambda\left(P_{n-1}(t) - P_n(t)\right) \\ \dfrac{dP_0}{dt} = -\lambda P_0(t) \Rightarrow P_0(t) = C\cdot e^{-\lambda t} \Rightarrow C = 1 \\ \qquad\qquad P_0(0) = 1 \end{cases}$$

$$\frac{dp_1}{dt} = \lambda P_0(t) - \lambda P_1(t) = \lambda\cdot e^{-\lambda t} - \lambda P_1(t)$$

$$P_1' + \lambda P_1 - \lambda e^{-\lambda t} = 0$$

$$P_1^{\text{одн}} = T(t)\cdot e^{-\lambda t}$$

$$T'\cdot e^{-\lambda t} - \lambda\cdot T\cdot e^{-\lambda t} + \lambda T\cdot e^{-\lambda t} - \lambda\cdot e^{-\lambda t} = 0$$

$$T' = \lambda \Rightarrow T = \lambda t + A$$

Отже $P_1(t) = (\lambda t + A)\cdot e^{-\lambda t}$

$P_1(0) = 0 \Rightarrow A = 0$

0 отне: $P_1(t) = \lambda t \cdot e^{-\lambda t}$

$\dot{P}_2 = \lambda^2 t \cdot e^{-\lambda t} - \lambda P_2$

$\dot{P}_2 + \lambda P_2 - \lambda^2 t \cdot e^{-\lambda t} = 0$

$P_2^{ogu} = T(t) \cdot e^{-\lambda t}$

$T' \cdot e^{-\lambda t} - \lambda \cancel{T} \cdot e^{-\lambda t} + \lambda T \cdot e^{-\lambda T} - \lambda^2 t e^{-\lambda t} = 0$

$T' = \lambda^2 T \Rightarrow T = \frac{\lambda^2}{2} t^2 + A$

$P_2 = \left(\frac{\lambda^2}{2} t^2 + A\right) e^{-\lambda t}$

$P_2(0) = 0 \Rightarrow A = 0$

$P_2(t) = \frac{\lambda^2}{2} t^2 \cdot e^{-\lambda t}$

$P_n(t) = \frac{\lambda^n}{n!} \cdot t^n \cdot e^{-\lambda t}$ – розподіл Пуас.

---

1) $f(x) = C(x^2 + B x^4) \cdot e^{-x^2/a^2}$ | $\langle \Delta x \rangle$ – ? C – ?

$\int_{-\infty}^{+\infty} f(x) dx = C\left(\int x^2 e^{-\frac{x^2}{a^2}} dx + B \cdot \int x^4 \cdot e^{-x^2/a^2} dx\right) = C\left(\frac{\Gamma(\frac{3}{2})}{a^{-3/2}} + B \cdot \frac{\Gamma(\frac{5}{2})}{a^{-5/2}}\right) = 1$

$\langle \Delta x^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2 = C\left(\int x^4 e^{-\frac{x^2}{a^2}} dx + \int x^6 e^{-\frac{x^2}{a^2}} dx\right) = \ldots$

$\langle x \rangle = 0$

$f(x) = f(-x)$

Б. 8.

1. знайти $\langle(\Delta E)^2\rangle$ для гарм. осц. однонідн. з ω_0 при темпе-ратурі T.

2. Зн. середньоквадр. флуктуацію z-вої компоненти маг. мом. елемент. компенент. h. Поле H нап-кому вздовж. осі z.

$$E = \mu H$$

$$\langle E^2\rangle = \sum_n \frac{E_n^2}{Z}\cdot e^{-\frac{E_n}{kT}} = \left|\frac{1}{kT} = \alpha\right| =$$

$$= \sum_n \frac{E_n^2}{Z}\cdot e^{-\alpha E_n} = \sum_n \frac{1}{Z}\frac{\partial^2}{\partial\alpha^2}\cdot e^{-\alpha E_n} =$$

$$= \frac{1}{Z}\frac{\partial^2}{\partial\alpha^2}\sum e^{-\alpha E_n} = \frac{1}{Z}\frac{\partial^2}{\partial\alpha^2} Z = \Big| \ldots \frac{\partial^2}{\partial T^2}$$

$$\frac{\partial}{\partial\alpha} = \frac{\partial T}{\partial\alpha}\cdot\frac{\partial}{\partial T}$$

$$\langle(\Delta E)^2\rangle = \langle E^2\rangle - \langle E\rangle^2$$

$$\langle(\Delta\mu)^2\rangle = \frac{\langle(\Delta E)^2\rangle}{H}$$

[ст. 33 (8)]

Б.14

① $C_p - C_V - ?$

$F = F(T,V)$

$S = -\left(\frac{\partial F}{\partial T}\right)_V$ $\qquad p = -\left(\frac{\partial F}{\partial V}\right)_T$

$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$ $\qquad C_V = T\left(\frac{\partial S}{\partial T}\right)_V$

$\left(\frac{\partial S}{\partial T}\right) = \left(-\left(\frac{\partial F}{\partial T}\right)_V\right)'_p$

---

② Част. рух. дискретно з кроком a.

$-a \quad 0 \quad a \quad 2a \quad 3a \longrightarrow$ Час дискретний з кроком Δt

Зн. ймов., що через час $T = const$ $x < 0$, $x(0) = 0$.

Ймов. руху вліво на a вдвічі менша ймов. руху вправо.

$P_l = \frac{1}{4}$, $P_r = \frac{3}{4}$

Кроків p_1 - вліво, p_2 - вправо

$k(P_l)^{p_1} \cdot (P_r)^{p_2}$

Σ-кількі : $N_0 = \frac{N}{2} + 1$

p - нечетна $\quad N_0 = \frac{N+1}{2}$

$N_1 = N_0 \dots N$

$$p = \sum_{p_1 = p_0}^{p}{}' K (p_e)^{p_1} \cdot (p_r)^{p_2}, \quad K = C_N^{N_1} = \frac{N!}{p_1! \cdot p_2!}$$

Б. 4

1) $F = -g_0 T \cdot \ln VT^2$; $\frac{C_p}{C_V}$ – ?

$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$; $C_V = T\left(\frac{\partial S}{\partial T}\right)_V$

$S = -\left(\frac{\partial F}{\partial T}\right)_V = g_0 \frac{\partial}{\partial T}(T \cdot \ln VT^2) = g_0 \cdot \frac{\partial}{\partial T} \cdot$
$\cdot \left(T(\ln V - 2\ln T)\right) = g_0 \left(\ln VT^2 + T\frac{2}{T}\right) =$
$= g_0 (\ln V \cdot T^2 + 2)$

$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = g_0 \cdot T \cdot \frac{2}{V}$

$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$

p – не стану: $p = -\left(\frac{\partial F}{\partial V}\right)_T = g_0 \frac{T}{V}$

$V = g_0 \cdot \frac{T}{p}$; $S = g_0 \left(\ln\left(\frac{g_0 T^3}{p}\right) + 2\right)$

$C_p = T\left(\frac{\partial S}{\partial T}\right)_p = 3 g_0$

$\frac{C_p}{C_V} = \frac{3}{2}$

2) Знайдіть внесок в C_V оберт. руху в двоат. молекули

$Z = Z_{пост.} \cdot Z_{оберт.} \cdot Z_{кол.}$

$$E_{\text{оберт.}} = \frac{p_\theta^2}{I^2} + \frac{p_\varphi^2}{2\mu r^2 \sin^2\theta}; \quad \mu r^2 = I$$

$$Z = \iint e^{-\left(\frac{p_\theta^2}{kT 2I} + \frac{p_\varphi^2}{2I \cdot \sin^2\theta \cdot kT}\right)} \cdot \frac{d\theta\, d\varphi\, dp_\theta\, dp_\varphi}{(2\pi\hbar)^2} =$$

$$= \frac{2\pi}{(2\pi\hbar)^2} \int_0^\pi \sqrt{2I\, kT\pi} \cdot \sqrt{2I \sin^2\theta \cdot kT \cdot \pi} \cdot d\theta =$$

$$= \frac{2\pi \cdot 2\pi I \cdot k \cdot T}{(2\pi\hbar)^2} = \frac{2I \cdot kT}{\hbar^2} \quad \left(\text{клас вин.}\right)$$

$$\ln Z = \ln\left(\frac{2}{\hbar^2} IkT\right)$$

$$S = \frac{\partial}{\partial T}\left(kT \ln \frac{2IkT}{\hbar^2}\right) = k \cdot \ln \frac{2IkT}{\hbar^2} + $$

$$+ k\cancel{T} \cdot \frac{1}{\cancel{T}}$$

$$c_V = T\left(\frac{\partial S}{\partial T}\right)_V = T \cdot k \cdot \frac{\partial}{\partial T}\left(\ln \frac{2IkT}{\hbar^2}\right) =$$

$$= T \cdot k \cdot \frac{1}{T} = k$$

11

Б.20

① Физ. объем, энтроп. вещ. ф-лою

$$S = C + A\ln(VT^3)$$

$$\Delta p - ?$$

$$S = C + A\ln V + 3A\ln T$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{\Delta p}{\Delta T}\Big|_{V=const}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \frac{B}{V} \Rightarrow \frac{\Delta P}{\Delta T} = \frac{B}{V};\quad \Delta p = \frac{B\cdot \Delta T}{V}$$

② $$\langle \Delta E^4\rangle = \langle (E-\bar E)^4\rangle = \langle E^4 - 4E^3\bar E + 6E^2\bar E^2 - 4E\cdot \bar E^3 + \bar E^4\rangle = \langle E^4\rangle - 4\langle E^3\rangle\cdot\langle E\rangle + 6\langle E^2\rangle\cdot\langle E\rangle^2 \underbrace{- 4\langle E\rangle\langle E\rangle^3}_{-4\langle E\rangle^4} + \langle E\rangle^4$$

$$f(E) = \frac{1}{e^{\frac{E-\mu}{kT}} - 1}$$

$$\langle E\rangle = \sum_k{}' E_k f(E_k) = \underbrace{\frac{E}{e^{\frac{E-\mu}{kT}} - 1}}_{=A}$$

$$\langle E^2\rangle = \frac{E^2}{A}$$

$$\langle E^3\rangle = \frac{E^3}{A}$$

$$\langle E^4 \rangle = \frac{E^4}{A}$$

$$\langle E \rangle^2 = \frac{E^2}{\left(e^{\frac{E-\mu}{kT}} - 1\right)^2}$$

$$\langle \Delta E^4 \rangle = \frac{E^4}{A_4} - 4\frac{E^3}{A} \cdot \frac{E}{A} + 6\frac{E^2}{A} \cdot \frac{E^2}{A^2} -$$

$$- 3\frac{E^4}{A^4} = \frac{E^4}{A} \cdot \left[1 - \frac{4}{A} + \frac{6}{A^2} - \frac{3}{A^3}\right]$$

При $\frac{E_k - \mu}{kT} \gg 1$, $e^{\frac{E-\mu}{kT}} \gg 1$

$$\langle \Delta E^4 \rangle = E^4 \cdot B - 4 \cdot B + 6B^2 - 3B^2$$

$= \cancel{\frac{E^4}{A}}$ где $B = e^{\frac{\mu - E}{kT}}$

№ 6.

1) $\langle(\Delta n)^2\rangle$ — ? для Пуассона

$$p_n = \sum_{n=0}^{\infty} \frac{\lambda^n t^n}{n!} e^{-\lambda t}; \quad \langle\Delta n^2\rangle = \overline{n^2} - (\bar{n})^2$$

$$\bar{n} = \sum_{n=0}^{\infty} n \cdot p_n = \sum_{n=0}^{\infty} \frac{\lambda^n t^n}{(n-1)!} e^{-\lambda t} = \lambda t \cdot$$

$$\cdot \sum_{n=1}^{\infty} \frac{(\lambda t)^{n-1}}{(n-1)!} e^{-\lambda t} = \lambda t \cdot e^{-\lambda t} \cdot e^{\lambda t} = \lambda t$$

$$(\bar{n})^2 = \lambda^2 t^2$$

$$\overline{n^2} = \sum_{n=0}^{\infty} n^2 p_n = \sum_{n=0}^{\infty} n^2 \cdot \frac{\lambda^n t^n}{n!} e^{-\lambda t} \cdot \sum_{n=0}^{\infty} n \cdot$$

$$\cdot \frac{\lambda^n t^n}{(n-1)!} = \lambda t \cdot e^{-\lambda t} \cdot \sum_{n=0}^{\infty} \frac{n \cdot (\lambda t)^{n-1}}{(n-1)!} = \lambda t \cdot e^{-\lambda t} \cdot$$

$$\cdot \sum_{n=0} \frac{d}{d(\lambda t)} \left(\frac{\lambda^n t^n}{(n-1)!}\right) = \lambda t \cdot e^{-\lambda t} \cdot \frac{d}{d(\lambda t)} \lambda t$$

$$\cdot \sum_{0}^{\infty} \frac{(\lambda t)^{n-1}}{(n-1)!} = \lambda t \cdot e^{-\lambda t} \frac{d}{d(\lambda t)} \lambda t \cdot e^{\lambda t} =$$

$$= \lambda t \cdot e^{-\lambda t} (e^{\lambda t} + \lambda t e^{\lambda t}) = (\lambda t + \lambda^2 t^2) \cdot$$

$$\cdot e^{\lambda t} = \lambda t + \lambda^2 t^2$$

$$\langle \Delta n^2 \rangle = \lambda^2 t^2 + \lambda t - \lambda^2 t^2 = \underline{\underline{\lambda t}}$$

(2) $\langle \Delta S \cdot \Delta p \rangle - ?$ при T, V

$$\Delta p = \left(\frac{\partial p}{\partial T}\right)_V \cdot \Delta T + \left(\frac{\partial p}{\partial V}\right)_T \cdot \Delta V$$

$$\Delta S = \left(\frac{\partial S}{\partial T}\right)_V \cdot \Delta T + \left(\frac{\partial S}{\partial V}\right)_T \cdot \Delta V$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{C_V}{T} \; ; \; \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\langle \Delta p \cdot \Delta S \rangle = \left\langle \left(\left(\frac{\partial p}{\partial T}\right)_V \cdot \Delta T + \left(\frac{\partial p}{\partial V}\right)_T \cdot \Delta V \right) \cdot \left(\left(\frac{\partial S}{\partial T}\right)_V \cdot \Delta T + \left(\frac{\partial S}{\partial V}\right)_T \cdot \Delta V \right) \right\rangle = \left\langle \left(\frac{\partial p}{\partial T}\right)_V \cdot \left(\frac{\partial S}{\partial T}\right)_V \cdot \Delta T^2 + \left(\frac{\partial p}{\partial T}\right)_V \cdot \left(\frac{\partial S}{\partial V}\right)_T \cdot \Delta T \cdot \Delta V + \left(\frac{\partial S}{\partial T}\right)_V \cdot \left(\frac{\partial p}{\partial V}\right)_T \cdot \Delta T \cdot \Delta V + \left(\frac{\partial p}{\partial V}\right)_T \cdot \left(\frac{\partial S}{\partial V}\right)_T \cdot \Delta V^2 \right\rangle = \left(\frac{\partial p}{\partial T}\right)_V \cdot \left(\frac{\partial S}{\partial T}\right)_V \cdot \frac{kT^2}{C_V} - \left(\frac{\partial p}{\partial V}\right)_T \cdot \left(\frac{\partial S}{\partial V}\right)_T \cdot \frac{kT}{\left(\frac{\partial p}{\partial T}\right)_V} = \left(\frac{\partial S}{\partial V}\right)_T \cdot \left(\frac{\partial S}{\partial T}\right)_V \cdot \frac{kT^2}{C_V} - \left(\frac{\partial S}{\partial V}\right)_T \cdot \left(\frac{\partial S}{\partial V}\right)_T \cdot \frac{kT}{\left(\frac{\partial S}{\partial V}\right)_T} = \left(\frac{\partial S}{\partial V}\right)_T \cdot \frac{C_V}{T} \cdot \frac{kT^2}{C_V} - \left(\frac{\partial S}{\partial V}\right)_T \cdot kT = 0$$

Б. 17.

(1) $\langle \Delta T \cdot \Delta p \rangle$ — T, V — незалежні для ідеального газу.

$$\left\langle \left(\left(\frac{\partial p}{\partial T}\right)_V \cdot \Delta T + \left(\frac{\partial p}{\partial V}\right)_T \cdot \Delta V \right) \Delta T \right\rangle =$$

$$= \left\langle \left(\left(\frac{\partial p}{\partial T}\right)_V \cdot \Delta T^2 + \underbrace{\left(\frac{\partial p}{\partial V}\right)_T \Delta V \cdot \Delta T}_{=0} \right) \right\rangle =$$

$$= \left(\frac{\partial p}{\partial T}\right)_V \cdot \langle \Delta T^2 \rangle = \left(\frac{\partial p}{\partial T}\right)_V \cdot \frac{kT^2}{C_V} =$$

$$= \left(\frac{\partial S}{\partial V}\right)_T \cdot \frac{kT^2}{C_V}$$

10.

(1) Зн. флукт. угл. зерк. гальв. при зад. T. Модуль кручен. кручения $= \alpha$. Пружина эн. $E = \frac{\alpha\varphi^2}{2}$

Розв.: $U = \frac{\alpha\varphi^2}{2}$

$$w(x) \sim \exp\left\{\frac{\Delta p \Delta V - \Delta S \Delta T}{2kT}\right\}$$

$$\Delta p \cdot \Delta V \to \Delta A \Delta a$$

$$\partial A = -A da$$

$$A = -\frac{\partial A}{\partial a} = \frac{\partial U}{\partial \varphi} = -\alpha\Delta\varphi$$

$$w(x) \sim \exp\left\{-\frac{\alpha\Delta\varphi \cdot \Delta\varphi}{2kT}\right\} = \exp \cdot$$

$$\cdot \left\{\left(\frac{\alpha}{2kT}\right)^{-1} \Delta\varphi^2\right\}$$

$$\langle \Delta\varphi^2 \rangle = \left(\frac{\alpha}{kT}\right)^{-1}$$

Б.

① $W(x)$ - ? зн. розподіл, якщо ентропія за Больцманом max і $\langle x^2 \rangle = a^2$

$$\overline{W(x)} \sim e^{S(x)}$$

$$\Delta x = x - \overline{x}, \quad \overline{x} = 0$$

$$\Delta S = S(0) - S(x)$$

$$S(x) = S(0) + \frac{1}{2}\frac{\partial^2 S}{\partial x^2}\cdot \Delta x^2 + \ldots$$

$$W(x) \sim e^{\frac{1}{2}\frac{\partial^2 S}{\partial x^2}\Delta x^2}$$

$$W(x) \sim e^{\frac{1}{2}\alpha \Delta x^2}$$ Розподіл. Гауса

$$\langle x^2 \rangle = a^2$$

$$W(x) = \frac{1}{\sqrt{2\pi}\cdot a}\cdot \exp\left(-\frac{x^2}{2a^2}\right)$$

Б. 22.

① $\langle (\Delta u)^3 \rangle = \langle (u - \bar{u})^3 \rangle = \langle u^3 - 3u^2 \cdot \bar{u} + 3u \cdot \bar{u}^2 - \bar{u}^3 \rangle = \langle u^3 \rangle -$

$- 3 \langle u^2 \rangle \cdot \langle u \rangle + 3 \langle u \rangle^3 - \bar{u}^3 =$

$= \langle u^3 \rangle - 3 \langle u^2 \rangle \cdot \langle u \rangle + 2 \langle u \rangle^3$

$$\langle u \rangle = \int_{u_1}^{u_2} \frac{1}{u_2 - u_1} u \, du = \frac{u_2^2 - u_1^2}{2(u_2 - u_1)} =$$

$$= \frac{u_1 + u_2}{2}$$

$$\langle u^2 \rangle = \int_{u_1}^{u_2} u^2 \, du = \frac{1}{(u_2 - u_1)^2} \int_{u_1}^{u_2} du$$

. . .

$$2.\quad Z(T) = \sum_n e^{-E_n/kT}$$

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right) \qquad \sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q}$$

$$F = -kT \ln Z$$

$$Z(T) = \sum e^{\hbar\omega(n+\frac{1}{2})/kT} = e^{\hbar\omega/2kT}$$

$$= \sum_{n=0}^{\infty} e^{\hbar\omega\,0/2kT} \cdot e^{n\hbar\omega/kT} = \frac{e^{0}}{1-e^{\hbar\omega/kT}}$$

Б. 13.

1 $\mathcal{H}(p,q) = \frac{p^2}{2m}$

$Z_{1\,\text{тр}} = \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} V$

$Z_N = \frac{1}{N!} \cdot Z_1^N$

$S = -kT \ln Z$

N ?

② Знайти теплоємність одноат. газу частинок, що знах в долоному V при T

$$Z_N = \frac{(Z_1)^N}{N!} \text{ — ган. стат. сума.}$$

$$Z_1 = \iint_{-\infty}^{\infty} e^{-\frac{H(p,q)}{kT}} dp\,dq =$$

$$= V \int_0^{\infty} e^{-\frac{p^2}{2mkT}} dp = V \cdot \frac{1}{2}\sqrt{\pi \cdot 2 m \cdot k \cdot T} = N \cdot T^{1/2} \sim$$

$$\sim A \cdot T^{1/2}$$

$$Z_N = \frac{1}{N!} A T^{1/2}$$

$$C_V = \left(\frac{\partial S}{\partial T}\right)_V \qquad S = k \ln Z_N$$

① $W(x)$ $\qquad \langle x^2 \rangle = a^2$

7/ $\langle \Delta T \Delta P \rangle$ - через T, V

$$\Delta P = \left(\frac{\partial P}{\partial T}\right)_V \Delta T + \left(\frac{\partial P}{\partial V}\right)_T \Delta V$$

$$\langle \Delta P \Delta T \rangle = \left\langle \left(\frac{\partial P}{\partial T}\right)_V \Delta T^2 + \left(\frac{\partial P}{\partial V}\right)_T \Delta V \Delta T \right\rangle$$

$$= \left(\frac{\partial P}{\partial T}\right)_V \langle \Delta T^2 \rangle = \left(\frac{\partial P}{\partial T}\right)_V \frac{kT^2}{C_V} =$$

$$= \left(\frac{\partial S}{\partial V}\right)_T \cdot \frac{kT^2}{C_V}$$

1. Задано F — вільна енергія

Знайти $C_p - C_v$

$$-\left(\frac{\partial F}{\partial T}\right)_V = +S, \quad -\left(\frac{\partial F}{\partial V}\right)_T = p$$

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p, \quad C_v = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$F = F(T, V)$$

$$\frac{\partial S}{\partial T} = \left(-\left(\frac{\partial F}{\partial T}\right)_V\right)'_p$$

2. Частинка рухається дискретно з кроком a

Час дискретний з кроком Δt

$-a \quad 0 \quad a \quad 2a \quad 3a \quad x$

Знайти ймовірність, що через час $T = n\Delta t$ $x < 0$.

$x(0) = 0$

Ймовірність руху вліво на a втричі менша ймовірності руху вправо

$$p_l = \frac{1}{4}, \quad p_r = \frac{3}{4}$$

кроків N_1 — вліво, N_2 кроків вправо

$$K (p_l)^{N_1} (p_r)^{N_2}, \quad N_1 > N_2$$

N — парне $\quad N_0 = \frac{N}{2} + 1$

N — непарне $\quad N_0 = \frac{N+1}{2}$

$$N_1 = N_0 \ldots N$$

$$p = \sum_{N_1 = N_0}^{\omega} K (p_l)^{\omega} \cdot (p_r)^{N_2}, \quad K = C_N^{N_1} = \frac{N!}{N_1!\, N_2!}$$

(4.) $\langle (\Delta S)^2 \rangle$ – ? для ідеального газу. У змінних S, p.

а) $\Delta W = \exp\left\{\frac{\Delta p \Delta V - \Delta T \Delta S}{2kT}\right\}$ – гол. формула терм. фл.

$$\begin{cases} \Delta V = \left(\frac{\partial V}{\partial S}\right)_p \Delta S + \left(\frac{\partial V}{\partial p}\right)_S \Delta p \\ \Delta T = \left(\frac{\partial T}{\partial S}\right)_p \Delta S + \left(\frac{\partial T}{\partial p}\right)_S \Delta p \end{cases}$$

$f(x,y)$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} \cdot dy$$

$f(x_1 \ldots x_n)$

$$df = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right) \cdot dx_i$$

$$\Delta W = \exp\left\{\frac{1}{2kT}\left(\left(\frac{\partial V}{\partial S}\right)_p \Delta S \cdot \Delta p + \left(\frac{\partial V}{\partial p}\right)_S (\Delta p)^2 + \right.\right.$$

$$\left.\left. + \left(\frac{\partial T}{\partial S}\right)_p (\Delta S)^2 - \left(\frac{\partial T}{\partial p}\right)_S \Delta S \cdot \Delta p\right)\right\}$$

$$\Delta W = \exp\left\{\frac{1}{2}\left(-\frac{(\Delta A)^2}{\langle \Delta A^2 \rangle}\right) - \frac{(\Delta B)^2}{\langle \Delta B^2 \rangle} \quad \ldots \right\}$$

$$\Delta W = \exp\left(\frac{1}{2}\left(\left(\frac{\partial V}{\partial p}\right)_S \frac{(\Delta p)^2}{kT} + \frac{\Delta S \cdot \Delta p}{kT}(\ldots) - \frac{(\Delta S)^2}{\frac{kT}{\left(\frac{\partial T}{\partial S}\right)_p}}\right)\right)$$

$$\frac{kT}{\left(\frac{\partial T}{\partial S}\right)_p} = \langle \Delta S^2 \rangle$$

$$\langle \Delta S^2 \rangle = \frac{kT}{\left(\frac{\partial T}{\partial S}\right)_p}$$

б) $\langle \Delta T^2 \rangle$ – ? S, p

$$\begin{cases} \Delta S = \left(\frac{\partial S}{\partial V}\right)_T \Delta V + \left(\frac{\partial S}{\partial T}\right)_V \Delta T \\ \Delta p = \left(\frac{\partial p}{\partial V}\right)_T \Delta V + \left(\frac{\partial p}{\partial T}\right)_V \Delta T \end{cases}$$

$$\Delta W = \exp\left\{ \frac{1}{2kT}\left(\left(\frac{\partial p}{\partial V}\right)_T (\Delta V)^2 + \left(\frac{\partial p}{\partial T}\right)_V \Delta T \Delta V - \left(\frac{\partial S}{\partial V}\right)_T \Delta T \Delta V - \left(\frac{\partial S}{\partial T}\right)_V \Delta T^2 \right) \right\}$$

$$\Delta W = \exp\left\{ \frac{1}{2}\left(- \frac{\Delta T^2}{\langle \Delta T^2 \rangle} \right. \right.$$

$$\langle \Delta T^2 \rangle = \frac{kT}{\left(\frac{\partial S}{\partial T}\right)_V} = \frac{kT^2}{T\left(\frac{\partial S}{\partial T}\right)_V} = \frac{kT^2}{C_V}$$

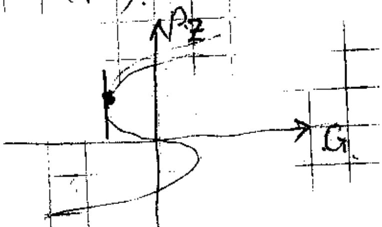
$$T\left(\frac{\partial S}{\partial T}\right)_V \equiv C_V$$

(3) Знайти умови існування одинарної петлі гістерезису для фазових переходів I роду.

Д.Т.С.:
Терм. пот. Гібса:

$$G(p_z) = G_0 + E_z p_z + a(T - T_c) p_z^2$$

$$\frac{\partial G(p_z)}{\partial p_z} = 0$$ – умова існування.



$$E_z + 2a(T - T_c) p_z = 0$$

$$\boxed{p_z = \frac{E_z}{2a(T_c - T)}}$$